Abstract

This paper studies the contracting problem of a firm which acquired another company and must reassign and incentivize the acquired employees. The firm would like to select high-ability employees for high-impact projects and incentivize them to exert unobservable effort. Each employee is privately informed about his ability, where a higher-ability employee has lower cost of effort. The resulting optimal employment contract creates two types of jobs: a “hybrid job,” where the employee splits time between high-impact and low-impact projects, and a job consisting of only one project. The results derive testable conditions under which the desired employee selection is achieved.

Keywords: optimal employment contracts, internal labor markets, adverse selection, moral hazard

JEL Classification Codes: D82, M52, G34.

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1 Introduction

A common way for a firm to expand is by acquiring another company. As part of this transaction, the acquiring firm usually receives a pool of new employees with heterogeneous abilities, the human capital of the acquired firm. It then faces the non-trivial and crucial challenge of integrating this pool of talent into the organization. In fact, recent evidence shows that a firm’s ability to reassign its acquired workers to new roles within the firm is a key factor in an acquisition’s success (Tate and Yang, 2016; Lee et al., 2018). Yet, we know little about how employment contracts must be designed in order to reassign and incentivize the new employees.

The challenge for the acquiring organization is two-fold. First, the acquired employees may have private information about their own ability or fit with the new firm’s projects, to which they may be reassigned.\footnote{For instance, the acquired employees, in many cases former entrepreneurs or employees of small start-ups, may have agreed to the acquisition because they no longer have the motivation or the ability to work on high-impact, innovative projects. Other literature (Van Wesep, 2010) studied compensation when the firm has more information than a potential employee about how well the two will match. We address the opposite case, when the employee has more information than the firm about his ability.} Second, these new employees must be given the right incentives to exert effort on their new projects. This implies a dual problem of selecting the “right” employees for the firm’s high impact projects and motivating these employees post-selection (Baker et al., 1988).

In this paper, we develop a model to analyze how organizations should optimally structure acquired employees’ assignment to projects and their compensation, given the dual challenge of generating truthful reporting and providing incentives for effort. The results derive conditions under which employee selection into high-impact projects can be achieved. They also provide theoretical foundations for features of employment contracts observed in practice: the use of base pay and options for high-impact projects, and contract provisions allowing employees to split their time between high-impact projects and unrelated low-impact work.

In our model, a risk-neutral firm runs two types of projects: a high-impact project the success of which depends on unobserved employee effort (e.g., R&D work which requires
effort towards innovation), and a low-impact project, which produces a fixed value and
does not require unobserved effort towards innovation (e.g., administrative work). The firm
faces two types of risk-neutral employees, who can be of either high or low ability. A high-
ability employee has a lower cost of supplying effort, and hence is more valuable to the firm.
However, neither employee ability nor effort can be directly observed by the firm. The firm
would like to assign a low-ability employee to the low-impact project, to assign a high-ability
employee to the high-impact project, and to incentivize the latter to exert effort.

The firm may offer contracts of the most general form, which means it can specify three
elements: First, the firm can specify the probability of assigning the employee to the high-
impact project (equivalent to specifying the fraction of time spent working on the high-
impact project versus the low-impact project). Second, if the employee is assigned to the
high-impact project, the firm can offer a bonus contingent on this project’s success. Third,
the firm may offer a fixed salary, which does not depend on the employee’s project or its
outcome. Throughout, we assume that the employee has limited liability.

We show that the optimal contract takes one of the following two forms: (i) the low-
ability employee is not assigned to the high-impact project at all, or (ii) both the low-ability
employee and the high-ability employee are assigned to high-impact projects, and one of
them has a higher probability of assignment. Hence, the employee splits his time between
high-impact (e.g., innovative) work and low-impact (e.g., administrative) work, i.e., receives
a “hybrid job”. Moreover, the employee with the higher assignment probability is offered
a lower bonus upon the success of the high-impact project. Furthermore, in case (ii), the
firm may offer the low-ability employee a positive fixed salary even though he could be
compensated more efficiently through an output-contingent bonus. This salary is an agency
cost arising endogenously, due to adverse selection in the internal labor market.

We derive the conditions under which each contract form is optimal. Throughout, our
measure of the firm’s value of assigning an employee to the high-impact project is given by
the summation of the firm’s profit and the employee’s compensation. The firm maximizes
this join surplus (the social value of the project) and pays the risk-neutral employee a rent.\footnote{We will show formally that social welfare is useful to characterize the shape of the optimal contract when the firm is constrained by having to achieve both selection and incentive provision (and thus has limited ability to change the rent by changing contracts).}

The contract of form (i) is optimal if the low-ability employee has a sufficiently high cost of effort and generates less social value by working on the high-impact project than by working on the low-impact project. It is then optimal for the firm to give this employee a fixed salary, in order to induce him to reveal his type and be selected out of the high-impact project. This result is related to the insights of the work on downsizing in the public sector (Jeon and Laffont, 1999). We complement these insights by showing when fixed payments are used to select out an employee \textit{before} assignment to the high-impact project.

The contract of form (ii) is optimal in all other cases. To arrive at this result, we first show that it is suboptimal to assign the high-ability employee to the high-impact project with both a higher probability and a higher bonus. In such a case, inducing the low-ability type to reveal his type would require paying this employee a suboptimally high fixed salary. Second, we examine which employee type is offered a high probability of assignment (and therefore a low bonus upon success). The firm has two tools to incentivize an employee to create social value from the high-impact project: the bonus for the high-impact project’s success (that incentivizes the employee to exert effort), and the probability of assigning the employee to the high-impact project (the probability with which the employee is given the chance to access the bonus). If the firm reduces the bonus, it can still generate the same expected social value by increasing the employee’s probability of assignment to the high-impact project. This trade-off is captured by the \textit{marginal rate of substitution (MRS) of bonus pay to probability of assignment} (holding fixed the expected social value from the contract and the employee’s ability). The trade-off can go one of two directions. In one case, the firm is more willing to sacrifice the incentive for effort (to lower the bonus) in exchange for selection (increasing the probability of assignment to the high-impact project) when it faces the high-ability employee (compared to when it faces the low-ability employee). This is the case in which the MRS decreases in employees’ ability. In the other case, the trade-off
goes in the opposite direction, and the MRS increases in employee’s ability. In either case, this trade-off implies that the optimal contract for at least one of the employees specifies probabilistic assignment to the high-impact project, resulting in a “hybrid job.” When the MRS decreases (increases) in employee type, the low-ability (high-ability) employee is offered a “hybrid job.”

Finally, we show that a fixed salary is offered only to the low-ability employee and only in certain cases. Paying a fixed salary is strictly worse for the firm than compensating the employee through the bonus for success, as a fixed salary does not incentivize effort. Thus, this salary is paid only if the firm faces a binding incentive compatibility constraint: if it increases the bonus and reduces the fixed salary for a certain ability type, the other ability type mimics. We find that this binding constraint may only emerge when the low-ability employee claims to be high ability. This happens only when there is a mismatch between the firm’s and the employee’s valuations of bonus payments versus probability of assignment to the high-impact project. Specifically, the firm’s MRS is increasing (decreasing) in the employee’s ability, but the low-ability employee is more (less) willing to sacrifice a higher probability of assignment in exchange for a higher bonus, compared to the high-ability employee. In other words, the employee’s MRS is decreasing (increasing) in the employee’s type. The firm can correct for this mismatch by offering the low ability employee a rent in the form of fixed salary.

The model speaks to the problem of re-assigning acquired human capital: acquiring firms usually have a fixed pool of new employees, and some employees are better fits than others for the new firm’s high-impact projects. Our results lead to a couple of main implications. First, we show that the key to separating employees is the availability of two types of valuable projects, only one of which requires unobservable effort. In fact, this is the crucial element that distinguishes our model and our results from the broader literature on optimal contracts.

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3 The insight that the low-ability employee may be assigned to the high-impact project with positive probability relates our paper to work on constraints to the efficient use of promotions (Ke et al., 2018) or the inefficient use of work requirements in conjunction with promotions (Barlevy and Neal, 2019). In a different setting (with dynamic moral hazard), Axelson and Bond (2015) also show how two contracts with different rewards and different probabilities of working on the high-impact task co-exist in equilibrium.
with adverse selection and moral hazard.\footnote{Sappington and Lewis (2000); Bernardo et al. (2001, 2008); Inderst and Klein (2007); Gottlieb and Moreira (2014); Chade and Swinkels (2016).} The existence of low-impact projects is a natural feature in organizations, and once this is taken into account, we depart from the previous results in the literature that the optimal policy is to offer the same contract to all employee types. For instance, Gottlieb and Moreira (2014), Sappington and Lewis (2000), and Chade and Swinkels (2016) assume only one type of project for the firm and show that no employee selection is optimal.\footnote{For the comparison with Chade and Swinkels (2016), see also footnote 19 in the main text.}

A second implication is that firms who face the problem of re-assigning heterogenous employees usually allow for “hybrid jobs.” Indeed, such jobs are common in firms that regularly acquire other smaller companies. For instance large technology companies commonly allow engineers to allocate a percentage of their time to working on innovative projects (which usually involve unobservable effort).\footnote{Google’s “20 percent rule” is a well-known example.} Such contracts are observed even though these companies also have dedicated R&D units, where employees are exclusively assigned to innovative work. Our model shows that having these two types of employment contracts co-exist is optimal for employee self-selection.

Our results highlight a key case when employee selection is preferable: heterogenous project-specific fit. While this is different from assuming intrinsic motivation on the part of the high-ability employee (Murdock, 2002; Benabou and Tirole, 2003; Besley and Ghatak, 2005), it leads to a related intuition: the high-ability type exerts more effort for the same pay, which makes his selection preferable.

The rest of the paper is organized as follows. Section 2 describes the model and presents several properties of the contracting problem. Section 3 provides three key benchmarks. Section 4 derives the optimal employment contract, and Section 5 discusses the results in the context of our application. Section 6 concludes and the Appendix contains the proofs.
2 The Model

We consider an environment with two players: a firm and an employee (who is already hired by the firm). The employee has one of two possible types, \( \theta \in \{ H, L \} \). This type is the employee’s private information, while the firm has prior belief \( \mu_\theta \) that the employee is of type \( \theta \). The employee must work on a project. This can be either an high-impact project, the outcome of which depends on the employee’s private (unobservable) effort towards innovation, or a low-impact (administrative) project which can be perfectly monitored by the firm produces a safe output with value \( W > 0 \).

The high-impact project has a set of possible outcomes given by \( Y = \{ 0, 1 \} \), where \( y \in Y \) is the value of the outcome, \( y = 1 \) denotes a success, and \( y = 0 \) denotes a failure. The probability of a success, \( q(e_\theta) \), depends linearly on the employee’s effort \( e_\theta : q(e_\theta) = q_0 + q_1 e_\theta \), with \( q_0, q_1 \in \mathbb{R}, q_0, q_1 \geq 0 \).\(^7\) The \( \theta \)-type employee has cost of effort \( c(\theta, e) \) for \( e \in \mathbb{R}_+ \). The cost function \( c(\theta, e) \) is continuously differentiable with respect to effort \( e \in [0, \frac{1 - q_0}{q_1}] \), \( \lim_{e \to 0} c_e(L, e) = 0 \), \( \lim_{e \to \frac{1 - q_0}{q_1}} c_e(H, e) \geq q_1 \), and

\[
c_e, c_{ee}, c_{eee} \geq 0.\(^8\)
\]

Moreover, \( c(H, e) \leq c(L, e) \), and the \( H \)-type has lower marginal cost of effort than the \( L \)-type: \( c_e(H, e) < c_e(L, e) \) for each \( e \in \mathbb{R}_+ \).

By the Revelation Principle, we can focus on the contracts in which the employee declares his true type. Depending on the declared type \( \theta \), the firm offers an employment contract \( C_\theta \). Since the firm and employee are both risk neutral, it is without loss of generality to focus on the following contract: a probability \( p_\theta \) of being assigned to a high-impact project, a bonus \( b_\theta \) paid out only in case of project success \( (y = 1) \), and a rent \( r_\theta \) that is the fixed salary received by the employee, non-contingent on project assignment or outcome.\(^9\) We assume

\(^7\)Note that, as long as \( q \) is concave, we can always re-measure effort such that linearity is obtained for a convex cost of effort \( c(\theta, e) \).

\(^8\)The subscripts \( e \) denote derivatives with respect to effort.

\(^9\)In particular, if \( \bar{b}_\theta(y) \geq 0 \) is the bonus after \( y \) and \( \bar{r}_\theta \geq 0 \) is the fixed salary, re-define \( \bar{b}_\theta = \bar{b}_\theta(1) - \bar{b}_\theta(0) \) and \( \bar{r}_\theta = \bar{r}_\theta + \bar{b}_\theta(0) \). It is clear that it is without loss to assume \( \bar{b}_\theta(1) \geq \bar{b}_\theta(0) \) since otherwise the effort would
that the employee has limited liability: \( b_\theta \geq 0 \) and \( r_\theta \geq 0 \ \forall \theta, y. \) 

**Timing of Actions.** The timing of actions is the following:

1. The employee declares his type \( \hat{\theta} \). On the equilibrium path, he reports his true type: \( \hat{\theta} = \theta \).

2. The employee is paid a fixed salary (a rent) \( r_\theta \).

3a. With probability \( p_\theta \), the firm assigns the employee to the high-impact project. After selection, the employee chooses private effort \( e_\theta \). The firm receives value \( y \in \{0, 1\} \) from the project, and it pays a bonus \( b_\theta \) (in addition to \( r_\theta \)) if \( y = 1 \). This is the end of the game.

3b. With probability \( 1 - p_\theta \), the firm assigns the employee to the low-impact project. The firm obtains a fixed value \( W \) and the employee receives no additional bonus.

As shown above, we assume that the employee’s type \( \theta \) is not observable (adverse selection), and that effort \( e_\theta \) is also not observable (moral hazard).

**Payoffs.** The total expected payoff for employee \( \theta \) from contract \( C_\theta \) is given by

\[
p_\theta \cdot V(\theta, b_\theta) + r_\theta,
\]

where

\[
V(\theta, b_\theta) \equiv q(e(\theta, b_\theta)) \cdot b_\theta - c(\theta, e(\theta, b_\theta))
\]

denotes the payoff given the optimal effort with bonus \( b_\theta \). Note that, given risk-neutrality, the fixed salary \( r_\theta \) does not depend on project assignment. The equilibrium effort \( e(\theta, b_\theta) \) for type \( \theta \) is determined by

\[
e(\theta, b_\theta) = \text{arg max}_e q(e) \cdot b_\theta - c(\theta, e).
\]

\(^{10}\text{Otherwise, “selling a project” to the employee, with a price accepted only by the } H\text{-type is optimal.}\)
Given the properties of the cost function \( c(\theta, e) \), it follows that \( e(H, b) > e(L, b) \), and \( e(H, b_H) < 1 \) (since it is suboptimal for the firm to offer \( b_\theta \geq 1 \)).

**The Firm’s Problem**

Upon assigning an employee of type \( \theta \) to the high-impact project, the firm obtains expected payoff \( \pi(\theta, b_\theta) = q(e(\theta, b_\theta))(1 - b_\theta) \). Defining the social welfare function as \( S(\theta, b_\theta) = q(e(\theta, b_\theta)) - c(\theta, e(\theta, b_\theta)) \), we can write

\[
\pi(\theta, b_\theta) = S(\theta, b_\theta) - V(\theta, b_\theta).
\] (5)

**Assumption 1** The following condition is satisfied: \( \max_{b_L} \pi(L, b_L) < W < \max_{b_H} \pi(H, b_H) \).

Assumption 1 implies that, with full information over the employee’s type, the firm prefers to assign the \( L \)-type to the low-impact project and to assign the \( H \)-type to the high-impact project (we relax this Assumption in Online Appendix G). Thus, the firm would like to select the “right” employee (type \( H \)) for the high-impact project.\(^{11}\)

The firm’s expected profit maximization problem given \( W \) is

\[
\bar{J}(W) = \max_{\{p_H, p_L, b_H, b_L, r_H, r_L\}} \mu_H [p_H \pi(H, b_H) + (1 - p_H) W - r_H] \\
+ \mu_L [p_L \pi(L, b_L) + (1 - p_L) W - r_L],
\] (6)

subject to\(^{12}\)

\[
p_H V(H, b_H) + r_H \geq p_L V(H, b_L) + r_L; \quad (IC_H)
\]

\[
p_L V(L, b_L) + r_L \geq p_H V(L, b_H) + r_H. \quad (IC_L)
\]

\(^{11}\)Note, however, that Assumption 1 does not imply \( p_L = 0 \) in the optimal contract. As will be seen in Proposition 4, \( p_L = 0 \) happens only if \( \max_{b_L} S(L, b_L) \leq W \). The social welfare is the correct measure for deciding whether \( p_L = 0 \) since the firm must pay a certain rent to the employee.

\(^{12}\)The definition of \( V(\theta, b) \) and \( \pi(\theta, b) \) implies (4).
The firm maximizes its payoff $\tilde{J}(W)$ subject to the two incentive compatibility constraints. Constraint $IC_H$ ensures that the $H$-type employee prefers the contract $(p_H, b_H, r_H)$ to the contract designed for the $L$-type employee, $(p_L, b_L, r_L)$. Similarly, constraint $IC_L$ ensures that the $L$-type employee prefers the contract $(p_L, b_L, r_L)$ to the contract designed for the $H$-type employee, $(p_H, b_H, r_H)$. Throughout the paper, we analyze the non-trivial case in which the firm assigns at least one of the employees to the high-impact project with positive probability, so $\max\{p_H, p_L\} > 0$.

**Remark 1** Instead of a probabilistic assignment, we can see $p$ as a fraction of time the employee spends for the high-impact project. Time flows from $[0, \infty)$. For each $t \in [0, \infty)$, given effort $e_t$, the success arrives according to a Poisson process with average arrival rate $q(e_t) \times p_t$, where $p_t$ is the fraction of time spent for the high-impact project. The flow cost of effort $e$ is $c(\theta, e) \times p_t$. Then, over time interval $[t, t + \Delta)$, the employee’s payoff is approximately $p_t q(e_t) b_t \Delta - p_t c(\theta, e_t) \Delta$.\(^\text{13}\) Hence, the employee’s value is proportional to $p_t V(\theta, b_t) + r_t$, where $r_t$ is the flow “fixed salary.”

### 2.1 Preliminaries

We begin by deriving several preliminary properties of objective (6) and their implications for the optimal contract. First, given Assumption 1, constraint $IC_L$ holds with equality, since otherwise the firm would benefit from reducing $p_L$ or $r_L$.

**Lemma 1** No fixed salary is offered to the $H$-type employee ($r_H = 0$), and one of the employee types is fully assigned to the high-impact project (either $p_L = 1$ or $p_H = 1$).

The firm only rewards the $H$-type in case of the project’s success. Intuitively, increasing the reward increases the effort provided by the employee. Since the employee is risk-neutral, the firm can incentivize the highest effort from the $H$-type by loading the entire reward on the bonus $b_H$. Moreover, this change makes the $L$-type less willing to claim to be an $H$-type

\(^{13}\)Here, we assume that $b_t$ is paid whenever a success happens.
since he has to pay a higher effort cost for a good outcome. Finally, at least one of the employees is always assigned to the high-impact project. Otherwise, “scaling up” \((p_\theta, b_\theta, r_\theta)\) to \((kp_\theta, b_\theta, kr_\theta)\) for each \(\theta\) with \(k > 1\) would increase the firm’s payoff.

**Lemma 2** *In the optimal contract, if the L-type employee is offered a positive assignment probability and fixed salary \((p_L > 0 \text{ and } r_L > 0)\), then constraints \(IC_L\) and \(IC_H\) bind.*

The fixed salary is a cost to the firm, and it does not incentivize effort. Thus, the firm offers a fixed salary only if incentive compatibility is otherwise not satisfied. If only one incentive constraint were not satisfied, the firm could correct that by changing the contract terms \(p_\theta\) and \(b_\theta\). If the fixed salary is ever used, it must therefore be the case that both incentive compatibility constraints bind. To show this result analytically, we begin by assuming \(IC_H\) is not binding and \(r_L > 0\). Then, the firm could reduce \(r_L\) and increase \(b_L\), keeping the payoff to the \(L\)-type constant. This change would increase the firm’s profit, since the \(L\)-type would exert more effort. Hence, the firm would be better off with \(r_L = 0\).

**Lemma 3** *It is the case that “\(b_H \geq b_L\) and \(p_H \leq p_L\)” or “\(b_H \leq b_L\) and \(p_H \geq p_L\).”*

Lemma 3 shows that using the fixed salary does not allow the firm to offer a contract with \(p_H > p_L\) and \(b_H > b_L\). To see why, consider the case in which such a contract menu were offered. For the \(L\)-type to truthfully reveal his type, his contract must have \(r_L > 0\). If the \(L\)-type is indifferent between the resulting two contracts, then this implies that the \(H\)-type is strictly better off from his own contract. This happens because the value of both assignment and bonus are higher for the \(H\)-type than for the \(L\)-type, while the value of the fixed salary is the same for both types. However, as shown in Lemma 2, the incentive compatibility constraints \(IC_H\) and \(IC_L\) must both be binding, or else the firm could improve on the contract menu by reducing \(r_L\) and increasing \(b_L\). That is, the fixed salary cannot solve the distortions due to adverse selection and moral hazard simultaneously.
3 Benchmarks

We start by establishing three benchmarks in order to highlight the key elements of our model. First, we describe the result when employee types are observable.

**Proposition 1** (Moral Hazard Only) With observable types, in the optimal contract, \( p_H = 1 \), \( b_H > 0, r_H = 0 \), and \( p_L = 0, b_L = r_L = 0 \).

If the firm could observe employee types, it would always assign the \( H \)-type to the high-impact project and compensate him contingent on project success, in order to incentivize effort. The firm would assign the \( L \)-type only to the low-impact project and, as this project requires no effort, the firm would not provide any incentives for effort. Finally, no fixed salary would be paid, since such a payment would simply be a loss to the firm.

Next, we consider the case in which effort is inconsequential to the project success. We assume type \( \theta \in \{H, L\} \) has the probability \( q_\theta \) of achieving outcome \( y = 1 \), independent of effort, with \( q_H > W > q_L \). The rest of the setup is the same as described in Section 2.

**Proposition 2** (Adverse Selection Only) If project success is not dependent on effort, in the optimal contract, \( p_H = 1 \) and \( p_L = b_H = r_H = r_L = 0 \).

The optimal contract in this case assigns the \( H \)-type employee to the high-impact project and the \( L \)-type employee to the low-impact project. It pays the employee 0 regardless of project assignment, since the firm derives no benefit from incentivizing effort.

Lastly, we consider the case where \( W = 0 \).

**Proposition 3** (No low-impact project) If \( W = 0 \), then the optimal contract is the same regardless of employee type: \( p_L = p_H = 1, b_L = b_H > 0, r_H = r_L = 0 \).

Without a low-impact project that is valuable to the firm, the optimal contract assigns both employees only to the high-impact project. Incentivizing the \( L \)-type to select out would come at a cost, without sufficient benefit to the firm. With both employee types assigned to the same project, the firm maximizes profits by offering the same high-powered incentives to
all employees (as implied by Lemma 3). The existence of a valuable (for the firm) low-impact project, which does not require unobservable effort, will prove critical for our main result that the firm can achieve employee self-selection through the employment contact.

In the first two benchmarks, the firm always assigns each employee type to a different project. In the third benchmark, it assigns all employees to the high-impact project. In all cases, the firm never pays a fixed salary. In the next section, we show how these results change when we combine moral hazard and adverse selection in an environment with two valuable project types.

4 The Optimal Contract

We proceed to derive the optimal employment contract. In our analysis, we focus on an endogenous object (described below as MRS), and we present our results in relationship to the endogenous variables $S(\theta, b)$ and $V(\theta, b)$. We make this choice because, as shown below, these objects are both economically relevant and econometrically measurable, unlike the exogenous parameters that compose them. Moreover, the intuition behind our results is best understood in relationship to these two values rather than the exogenous parameters.

4.1 Full Assignment to One Project

First, if the $L$-type is very unproductive in the high-impact project compared to the value of his work on the low-impact project, then the firm does not assign this type to the high-impact project:

Proposition 4 If $W \geq S(L, 1)$, then only the $H$-type employee is assigned to the high-impact project in the optimal contract: $r_L > 0$, $p_L = 0$, $p_H = 1$. Conversely, if $W < S(L, 1)$, then the $L$-type is assigned with positive probability ($p_L > 0$).

14 Varying the terms of the contract would allow an experimenter to partially identify $S$ (by varying $p$ and $b$) and $V$ (by also varying $r$), but unobservable effort precludes directly measuring $q$ and $c$. 

13
If \( W \geq S(L, 1) \), then the \( L \)-type is not assigned to the high-impact project. He instead offers a fixed salary that makes his contract as appealing as taking the contract designed for the \( H \)-type. Notice that by not assigning the \( L \)-type to the high-impact project, the firm loses at most \( S(L, 1) \) and gains \( W \) from his work on the low-impact project. Hence, the firm gains by assigning the \( L \)-type fully to the low-impact project. Conversely, when \( W < S(L, 1) \), consider the case when \( p_L = 0 \) (and \( p_H = 1 \) by Lemma 3). Then, \( IC_H \) is not binding since the \( H \)-type values the contract with \( p_H > 0 \) more than the \( L \)-type. Hence, it is incentive compatible to offer a new contract \( \left( \tilde{p}_L, \tilde{b}_L, r_L \right) = (\varepsilon, 1, r_L - \varepsilon V(L, 1)) \) with a small \( \varepsilon > 0 \). This new contract saves the firm \( \varepsilon V(L, 1) = \varepsilon S(L, 1) \) in the fixed salary. The firm has to give up \( W \) when it assigns the \( L \)-type to the high-impact project, so its net gain from this new contract is \( \varepsilon (S(L, 1) - W) > 0 \). This new contract would be a gainful deviation, showing that indeed \( p_L > 0 \) in the optimal contract.

### 4.2 Probabilistic Assignment to Projects

#### The Analogue to a Single-Crossing Condition

The result of Proposition 4 shows that, when the \( L \)-type is sufficiently productive \( (S(L, 1) > W) \), both employee types are assigned to the high-impact project with positive probability. To study this case, we find it illuminating to first consider the pairs of contracts which do not include a fixed salary: \( \Psi^0 \equiv \{(p, b, 0) : p \in [0, 1], b \in [0, 1]\} \). We examine under what conditions the solution to the firm’s problem is in this set. If the solution is not in this set, a fixed salary may be used to expand the set of implementable contracts.

Consider the problem for the \( \theta \)-type employee of selecting a contract when offered a choice between a pair of contracts from \( \Psi^0 \). Since these contracts must satisfy incentive compatibility, \( p_\theta \leq p_{\theta'} \) if and only if \( b_\theta \geq b_\theta \) for each \( \theta \neq \theta' \). The key to the employee’s choice therefore lies in how the employee values a gain in the probability of assignment to the high-impact project relative to a loss in the bonus for success.

**Definition 1** Given a particular \( p, b, \) and \( \theta \), the **Marginal Rate of Substitution (MRS)**
of $b$ for $p$ for the employee of type $\theta$ is defined as $p \frac{V_b(\theta, b)}{V(\theta, b)}$, and the MRS for the firm facing the employee of type $\theta$ is defined as $p \frac{S_b(\theta, b)}{S(\theta, b) - W}$.

Given a particular $(p, b)$, we say the employee’s (firm’s) MRS is \textit{increasing} in $\theta$ if
\[
\frac{V_b(H, b)}{V(L, b)} \geq \frac{S_b(H, b)}{S(H, b) - W} \geq \frac{S_b(L, b)}{S(L, b) - W}.
\]
We say the employee’s (firm’s) MRS is \textit{stable} if it is increasing in $\theta$ for each $p, b \in [0, 1]$, or it is decreasing in $\theta$ for each $p, b \in [0, 1]$. When it is stable, we simply say the employee’s (firm’s) MRS is increasing or decreasing.

For the employee, the payoff function from a contract with $r = 0$ is given by $p \cdot V(\theta, b)$. The MRS is then obtained by calculating the marginal rate of substitution between $b$ and $p$, keeping $p \cdot V(\theta, b)$ fixed. Since the $H$-type has a lower marginal cost of effort than the $L$-type, he obtains both higher marginal utility from the increase in the assignment probability ($V(H, b) \geq V(L, b)$) and higher marginal utility from the increase in the bonus ($V_b(H, b) \geq V_b(L, b)$). If the employee’s MRS is decreasing (increasing), then it means that the $H$-type values the former relatively more (less).

As stated in the above definition, the relevant payoff for the firm is $S(\theta, b) - W$. With transferable utility, the firm maximizes the net social welfare created by the employee, $p \cdot (S(\theta, b) - W)$, minus a rent paid to the employee. If the incentive constraint is binding and the firm cannot change the rent, then $p \cdot (S(\theta, b) - W)$ is the measure of the firm’s net profit. The $H$-type both exerts more absolute effort for the same bonus and supplies marginally more effort in response to an increase in the bonus. This means that the social welfare increases more when the firm increases the probability of assignment for the $H$-type as opposed to the $L$-type ($S(H, b) - W \geq S(L, b) - W$). At the same time, it also increases more when the firm increases the bonus to the $H$-type ($S_b(H, b) \geq S_b(L, b)$). If the firm’s MRS is decreasing (increasing), then the former effect is relatively more (less) important when the firm faces the $H$-type. That is, the firm values selecting the $H$-type relatively more than incentivizing effort from this type, compared to when the firm is facing the $L$-type.

We can use the stability of MRS to establish regularity conditions for our contracting problem. In particular, we can use the stability of the MRS to derive an analogue to single-crossing in this contracting environment. Specifically, we consider the inter-type single
Figure 1: Decreasing Employee MRS. The contract for type $H$ is fixed at some $C_H$, and the shaded area is the set of contracts that satisfy $IC_H$ and $IC_L$.

crossing condition: that the indifference curves of two types in the two-dimensional space $(p, b)$ intersect only once.

**Lemma 4** *The employee’s (firm’s) preferences in the contract space $\Psi^0$ satisfy single-crossing if the employee’s (the firm’s) MRS is stable.*

The implication of these lemmas for the employee’s preferences over $p$ and $b$ is illustrated in Figure 1, which graphs the indifference curves for each employee type in the case of decreasing MRS for the employee.

In order to streamline the analysis and highlight the mechanism behind the main result, we present the results for the case in which the employee’s MRS is decreasing, and the firm’s MRS is stable.\textsuperscript{15} The stability of the MRS is obtained under a broad set of cost functions $c(\theta, e)$,\textsuperscript{16} and examples are provided in the Online Appendix.

\textsuperscript{15}In the Online Appendix F, we provide the complementary analysis when the employee’s MRS is increasing or when the firm’s MRS is not stable.

\textsuperscript{16}In fact, Lemma 5 in the Appendix shows that stability of MRS corresponds to log-submodularity or log-supermodularity of the value functions $V(\theta, b)$ and $S(\theta, b) - W$. 

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The Optimal Contract with Probabilistic Assignment

Lemma 3 implies that the firm faces a trade-off between incentivizing effort post-selection (by offering \( b_H \geq b_L \)) and assigning the high-ability employee to the high-impact project (by offering \( p_H \geq p_L \)). The trade-off between these two is captured by the firm’s MRS. The next Proposition shows how this trade-off is resolved:

**Proposition 5** Suppose the employee’s MRS is decreasing and \( W < S (L, 1) \). The optimal contract has the following properties:

1. **(MRS Co-Movement)** If the firm’s MRS is also decreasing, then \( r_L = 0 \), \( b_L \geq b_H \), and \( 1 = p_H \geq p_L > 0 \).

2. **(MRS Counter-Movement)** If the firm’s MRS is increasing, then \( r_L \geq 0 \), \( b_H \geq b_L \), and \( 1 = p_L \geq p_H > 0 \). A fixed salary is part of the optimal contract \((r_L > 0)\) if and only if \( b_H \neq b_L \).

The mechanism behind this result consists of three components, and we discuss each of them in turn.

First, the relative bonuses and the relative probabilities of assignment are determined by the firm’s trade-off between selection and incentives (captured by the firm’s MRS). When the firm’s MRS is decreasing (increasing), the firm values selecting the \( H \)-type relatively more (relatively less) than incentivizing effort from this type, compared to when the firm is facing the \( L \)-type. Thus, if its MRS is decreasing, the firm’s trade-off between solving adverse selection and solving the incentives problem is resolved in favor of solving adverse selection. As a result, more weight is placed on the part of the contract that solves adverse selection (it selects the \( H \)-type more often, i.e., \( p_H \geq p_L \)) and less weight on the part of the contract that solves moral hazard (the \( H \)-type receives a lower bonus). This leads to \( p_H \geq p_L \) and \( b_L \geq b_H \). Conversely, if the MRS is increasing, the firm’s trade-off between solving adverse selection and solving the incentives problem is resolved in favor of solving the incentives problem. As such, more weight is placed on the part of the contract that
solves moral hazard (it offers higher \( b_H \)) and less weight on the part of the contract that solves adverse selection (it selects the \( H \)-type less often, i.e., \( p_H \leq p_L \)).

To see this more clearly, consider the following two extreme cases. At the first extreme, suppose that \( c(H, e) = \alpha_H \cdot e \) for all \( e \in [0, 1] \) and \( c(L, e) = \alpha_L \cdot e \) for all \( e \in [0, 1] \) with \( \alpha_H < \alpha_L \). In this case, the \( H \)-type exerts effort 1 if \( q_1 \cdot b_H \geq \alpha_H \) and the \( L \)-type exerts effort 1 only if \( q_1 \cdot b_L \geq \alpha_L \). Hence, there is no gain for the firm from offering \( b_H > b_L \), but there is a large gain from selecting the \( H \)-type. This leads to \( b_L > b_H \) and \( p_H > p_L \).

At the second extreme, suppose that \( c(H, e) = \alpha_H \cdot e \) for all \( e \in [0, 1] \) but \( c(L, e) \to \infty \) for all \( e \in [0, 1] \). In this case, the \( H \)-type exerts effort 1 if \( q_1 b_H \geq \alpha_H \), while the \( L \)-type does not exert effort. If the effort is not so important to generate a good outcome, the social welfare from the \( L \)-type and the social welfare from the \( H \)-type are close to each other. Hence, there is not much gain from selection and the firm would like to offer \( p_H < p_L \) and \( b_H = \alpha_H / q_1 > 0 = b_L \).

Analytically, when different contracts are offered to different types, the firm has two possible incentive compatible deviations that it could undertake: offer all employees the contract designed for the \( H \)-type, or offer all employees the contract designed for the \( L \)-type. If the firm offers all employees the contract designed for the \( H \)-type, this deviation is not profitable if and only if

\[
p_L (S (L, b_L) - V (L, b_L)) + (1 - p_L) W - r_L \geq p_H (S (L, b_H) - V (L, b_H)) + (1 - p_H) W.
\]

Then, given constraint \( IC_L \), such a deviation is not profitable only if

\[
p_L (S (L, b_L) - W) \geq p_H (S (L, b_H) - W).
\]

Similarly, offering both types the contract designed for the \( L \)-type is not profitable only if

\[
p_H (S (H, b_H) - W) \geq p_L (S (H, b_L) - W).
\]

\(^{17}\text{ Except if } b_L = 0, \text{ when the } L \text{-type is essentially paid only the fixed salary.}\)
Combining (7) and (8), we obtain

\[(S(L, b_L) - W) (S(H, b_H) - W) \geq (S(L, b_H) - W) (S(H, b_L) - W). \tag{9}\]

It follows that differentiated contracts when the firm’s MRS is decreasing are optimal only if \(b_L \geq b_H\).\(^{18}\) When the firm’s MRS is increasing, they are optimal only if \(b_H \geq b_L\).

Second, we explain why MRS counter-movement leads to a fixed salary (rent \(r_L > 0\)). Remember that both the firm and the \(H\)-type employee would ideally solve both the adverse selection problem (by only selecting the \(H\)-type employee) and the moral hazard problem (by offering a higher \(b_H\) than \(b_L\)). However, Lemma 3 tells us this is impossible, and they must choose between the two. When the firm and the employee MRS co-move, it means that both the firm and the \(H\)-type employee prefer to prioritize solving adverse selection (for MRS decreasing) or that they both prefer to prioritize solving moral hazard (for MRS increasing). So when the MRS is decreasing, the \(H\)-type finds an increase in the probability of assignment (the solution to adverse selection) relatively more valuable than the \(L\)-type, for the same decrease in the bonus. Thus, a contract with \(r_L = 0\), \(p_H \geq p_L\) and \(b_H \leq b_L\) is incentive compatible (see Figure 1). This is exactly the type of contract preferred by the firm. Hence, the optimal contract can be implemented without a fixed salary.

When the firm and the employee MRS counter-move, it means that the firm’s trade-off between selection and incentives goes in the opposite direction to the \(H\)-type employee’s trade-off. If the firm’s MRS is increasing, then the firm prioritizes incentivizing effort, and the differentiated contract preferred by the firm has \(p_H \leq p_L\) and \(b_H \geq b_L\). Without a fixed salary, this contract is not incentive compatible. To achieve incentive compatibility, the firm

\[\left( S(H, b_L) - W + \int_{b=b_L}^{b_H} S_b(H, b) \, db \right) (S(L, b_L) - W) \geq \left( S(L, b_L) - W + \int_{b=b_L}^{b_H} S_b(L, b) \, db \right) (S(H, b_L) - W) = S(H, b_H) - W \tag{10}\]

Since the firm’s MRS is decreasing, we have \(S_b(H, b_L) (S(L, b_L) - W) \leq S_b(L, b_L) (S(H, b_L) - W)\). Together with the identity \((S(H, b_L) - W)(S(L, b_L) - W) = (S(L, b_L) - W)(S(H, b_L) - W)\), we have (10) only if \(b_L \geq b_H\).
adds a rent $r_L \geq 0$ to the $L$-type’s contract.

Third, in contrast to our benchmarks, the firm can achieve employee self-selection through the employment contract. Crucial to this result is the existence of two kinds of valuable projects for the firm, high-impact and low-impact, where unobservable effort is important only in the former. In fact, the existence of the low-impact project with positive value for the firm ($W > 0$) is the key element that distinguishes our model and our results from the literature of optimal contracts with adverse selection and moral hazard.\(^\text{19}\)

The insight that comes from Proposition 5 may be summarized as follows. In an environment with both moral hazard and adverse selection, the firm cannot solve both problems simultaneously. It faces a trade-off between solving adverse selection and solving moral hazard. In either case, it can separate the employees by prioritizing solving the problem that brings it the higher marginal benefit (which we can measure by MRS). How costly it is to achieve separation depends, however, on how the employees value the solutions to the two problems, relative to one another. When the high ability (low cost) employee’s relative valuation of assignment (solving adverse selection) versus bonus pay (solving moral hazard) goes in the opposite direction to the firm’s, separation is costlier. The firm must pay the low ability employee an agency cost. This cost arises endogenously to compensate for the firm’s and the employees’ different relative valuations of selection versus incentives.

Finally, note that our main result is not limited to the specific case of two project outcomes or two employee types. The same result and its intuition carry over to settings with multiple employee types or multiple project outcomes (see Online Appendices C and D).

\(^{19}\)Chade and Swinkels (2016) offers a method of solving a contracting problem with adverse selection and moral hazard. Their decoupling method first solves a cost minimization problem given the principal’s value for each type, then solves the selection problem using this value. Finally, they verify that all the constraints are simultaneously satisfied. When the low-impact project is added, the principal can decide both assignment probabilities and bonuses. The problem thus becomes multi-dimensional, and their decoupling method does not work.
When is the MRS Decreasing or Increasing?

Consider first the firm’s trade-off. If the firm marginally increases the assignment probability by one unit, then the net social welfare changes by \( S(\theta, b) - W \) (a change along the extensive margin of absolute effort). If the firm increases the bonus marginally, then it changes the social welfare through the higher marginal effort it incentivizes (a movement along the intensive margin of marginal effort), since \( pS_b (\theta, b) = pq_1 e_b (\theta, b) \).

The above two observations imply two sufficient conditions under which the firm’s MRS is decreasing. One is that \( W \) is very close to \( S(L, b) \), such that the extensive margin for the \( L \)-type is very small. The other is that it is very important for the firm that the employee exerts some threshold level of effort, but effort beyond this threshold does very little to improve the probability of success.\(^{20}\) For example, if the high-impact project’s success depends on following a relatively rigid recipe known to everyone, then all employees might implement the same recipe. Then, in the optimal contract, both types of employees exert effort near the threshold. The gain from the extensive margin is large for the \( H \)-type since he exerts the target effort at a lower cost, while the gain from the intensive margin is very small for both types.

Next, consider the employee’s trade-off. If the firm increases the assignment probability marginally, then the employee’s value changes by \( V(\theta, b) \) (the gain on the extensive margin). If the firm increases the bonus, then it changes the employee’s value through the higher expected bonus and the marginal change in the effort provided by the employee (the intensive margin). Applying the Envelope Theorem, we can in fact ignore the latter effect. Then, the change is simply the probability of receiving the bonus, \( q_0 + q_1 e (\theta, b) \).

The employee’s MRS is decreasing if the extensive margin is more valuable for the \( H \)-type than the intensive margin (and vice versa for the \( L \)-type). In other words, for the \( H \)-type, having an opportunity to work on the high-impact project is more important than getting a more generous split of the output in case of success. The above observations imply that the

\(^{20}\)One may wonder if this condition contradicts the linearity assumption of \( q(e) \). We can re-measure \( e \) such that \( q(e) \) is linear and the condition is in terms of \( c(e) \): the marginal cost of effort is small until some threshold, but increases very rapidly when an agent exerts effort above the threshold.
employee’s MRS is decreasing when success in innovation is highly dependent on providing some threshold level of effort, but the marginal benefit of providing more effort is small. In this case, the extensive margin differs across types given different absolute costs of effort, while the intensive margin is similar across types.

Lastly, we examine when it is more likely to obtain MRS counter-movement, and hence a fixed salary as part of the optimal contract.

**Corollary 1** A fixed salary for the low-ability employee is part of the optimal contract if 
\[ \Delta e \equiv e(H, b) - e(L, b) \text{ is sufficiently small and } \Delta e_b \equiv e_b(H, b) - e_b(L, b) \text{ is sufficiently large.} \]

The two conditions capture the two reasons why we may obtain MRS counter-movement. First, \( W \) only affects the firm’s MRS and not the employee’s MRS. Second, when the firm increases the bonus \( b \), it increases the employee’s effort, and it also increases the employee’s share of the project’s output, given the effort. The former matters for social welfare, but it does not matter for the employee’s value (given the Envelope Theorem). Conversely, the latter does not matter for social welfare while it does for the employee’s value. In particular, consider the difference across employee types in the effect \( b \) has on social welfare, \( S_b(H, b) - S_b(L, b) \). It becomes larger when the difference across employee types in their reaction to the bonus becomes larger, i.e., when \( \Delta e_b \) is large. The higher the difference \( S_b(H, b) - S_b(L, b) \), the more likely the firm’s MRS is increasing in \( \theta \). By contrast, the difference across types in employees’ value, \( V(H, b) - V(L, b) \), becomes larger when there is a higher difference between types in the probability of receiving the bonus, i.e., when \( \Delta e \) is large. The higher the difference \( V(H, b) - V(L, b) \), the more likely the employee’s MRS is decreasing in \( \theta \).

An example for how these conditions play out comes from the following case. Once selected, each employee exerts approximately the same level of effort; however, the marginal cost to increase effort is very high for the \( L \)-type and not so high for the \( H \)-type. Thus, the reaction to the bonus is very different across types. We provide a numerical example with
MRS counter-movement in the Online Appendix E.\footnote{We also show that, since }\( e_\theta (\theta, b) = \int_{b_0}^{b} e_\theta (\theta, b') \, db' \), the condition for MRS counter-movement is relatively restrictive. This may explain why a fixed salary may be rarely used in practice. More details are provided in the Online Appendix.

**Characterization of the Optimal Contract**

When both the employee and the firm have a decreasing MRS, Lemma 1 and Proposition 5 imply that \( p_H = 1 \) and \( r_H = r_L = 0 \). Substituting constraint \( IC_L \) into (6), we obtain

\[
\max_{b_L \geq b_H \geq 0} \mu_H (\pi (H, b_H) - W) + \mu_L \frac{V (L, b_H)}{V (L, b_L)} (\pi (L, b_L) - W). \tag{11}
\]

**Proposition 6** With MRS co-movement, the optimal contract solves (11).

Next, consider the case in which there is MRS counter-movement. In this case, both \((IC_H)\) and \((IC_L)\) hold with equality, and substituting these constraints into (6), we obtain

\[
\max_{b_H \geq b_L \geq 0} \frac{V (H, b_L) - V (L, b_L)}{V (H, b_H) - V (L, b_H)} (\pi (H, b_H) - W) + \mu_L (\pi (L, b_L) - W - V (L, b_H) + V (L, b_L)). \tag{12}
\]

**Proposition 7** With MRS counter-movement, the optimal contract solves (12). Moreover, a fixed salary is offered \((r_L > 0)\) if and only if \( b_H > b_L \).

In the Online Appendix E, we provide examples for the MRS co-movement case and for the MRS counter-movement case.

**5 Applied Implications**

We discuss how our model can be used to derive implications for the type of employment contracts offered when both truthful reporting of one’s type and incentives provision are important concerns. This is relevant, for instance, in organizations that acquire other smaller
companies. A consequence of acquisitions is that the organization is faced with a fixed pool of new employees which it must assign to new roles. When designing an employment contract for these new employees, adverse selection matters because these employees may plausibly have private information as to their motivation or ability to pursue high-impact (for instance, innovative) work within the firm. Moreover, incentive provision matters because these employees must be motivated to work for the firm, and they are protected by limited liability. Our model allows us to analyze a key question for the above employment contracts: how job descriptions and compensation should be balanced in order to select the “right” employees for high-impact projects.

A first applied implication of the model is that employee self-selection requires that the organization offers two types of roles. Moreover, one these roles can be easily monitored, while the other is the one for which effort is unobservable and project-specific fit is important. Furthermore, these roles must be suitable for employees with the same observable characteristics (who only differ in their project-specific fit). These conditions point to employee self-selection working successfully in organizations with enough scale or complexity. For instance, these conditions may be met in organizations which have both managerial positions (with business development goals which require unobservable effort) and high-level technical positions (which do not require unobservable effort), open to employees with similar training or backgrounds. In smaller organizations, with less diversity of roles, employee self-selection may not be possible.

A second applied implication is that “hybrid jobs” achieve employee self-selection goals. The model’s results offer cues as to which types of employees should be offered such jobs. When there are no extreme differences in ability, it is optimal for the firm to offer all employees a positive probability of working on high-impact, innovative projects. The firm can effectively induce employees to self-select by offering two contracts. One contract assigns the employee to work on a high-impact, innovative project full time \( p = 1 \), but with lower rewards in case of success. This can be achieved, for instance, by assigning the employee to an R&D unit or an incubator inside the firm. Moreover, lower compensation requires
that, in such settings, the firm retains a high ownership share in any new product that is
developed. Another contract gives the employee a “hybrid job,” which splits the employee’s
time between work on an innovative project ($p < 1$) and “regular” work, but promises higher
rewards in case of success. This type of contract is used, for instance, by companies which
allow employees to use part of their time for innovative work outside of their usual job tasks.

The movement in the firm’s MRS when it faces one employee type versus another can
be used to discern which employees should be offered “hybrid jobs.” Firms with decreasing
MRS are the ones which prefer selecting more often a high-ability employee for the high-
impact projects. In practical terms, these are the firms for which the most value is expected
to be produced as long as some threshold absolute effort is exerted, and incentivizing effort
above that threshold is very costly. For instance, this may describe a firm which pursues
incremental innovation on its products, such as adding new features. Some threshold lab
effort is expected to deliver new features, but above that, marginal improvements on the
features are not as valuable. In this case, the firm does best by assigning full time to
one project those employees who will supply enough effort for the lowest compensation
(for instance, by setting up an incubator or a research unit where the employees are fully
dedicated to innovation). The intuition for how to select the employees in this case is closely
related to the intrinsic motivation literature, in that offering lower pay will select the right
employees, who are willing to exert enough effort even for low compensation.

Firms with increasing MRS are the ones which prefer offering high-powered incentives to
their high-ability employees, even if it implies assigning these employees less often to high-
impact projects. For these firms, achieving higher performance is valuable. For instance, this
could apply in the case of new product development or ground-breaking innovation. Then,
the “right” employees will prefer a “hybrid job”, like the “20 percent time,” that gives them
high rewards when they do succeed, e.g., higher ownership share in the product.
6 Concluding Remarks

In this paper, we present a model to study a fundamental problem for employment contracts: the joint problem of inducing truthful reporting by employees and providing incentives for effort post project assignment. We characterize the optimal employment contract in this environment. This provides a framework for studying the types of contracts offered by firms when both adverse selection and moral hazard are relevant concerns, as is the case in acquisitions focused on gaining a pool of talent for innovative work. Our findings show that the optimal contract is implemented by offering employees a menu, where each contract in the menu trades off the fraction of time the employee is expected to spend on innovative work against the employee’s monetary compensation. One type of contract assigns the employee fully to one project, for instance to an R&D unit or incubator within the firm, but offers lower monetary compensation. The other contract creates a “hybrid job,” with specifies a fraction of time to be spent on a high-impact project versus unrelated, low-impact work, but with the promise of high rewards for the high-impact project’s success. By showing the key role played by the firm’s and the employee’s MRS in shaping the optimal contract, we provide a measurable object that can be used in applied settings to determine which types of employees should be offered “hybrid jobs.” Finally, in the Online Appendix, we show that our results survive a generalization of the model to multiple project outcomes or multiple employee types. Thus, our insight about the use of “hybrid jobs” in environments with adverse selection and moral hazard extends to more complex organizations.
References


A Preliminaries

Although $\theta$ is binary, we assume that $H$ and $L$ are real numbers and all the functions are well defined for $\theta \in [L, H]$, so that we can take derivatives with respect to $\theta$. In the proofs, the following concavity/convexity results will be useful:

\[
e_\theta(\theta, b) = -\frac{c_{\theta e}(\theta, e(\theta, b))}{c_{ee}(\theta, e(\theta, b))};
\]
\[
e_{ab}(\theta, b) = -q_1 \frac{c_{\theta e}(\theta, e(\theta, b))}{[c_{ee}(\theta, e(\theta, b))]^2} + \frac{c_{\theta e}(\theta, e(\theta, b)) c_{\theta e}(\theta, e(\theta, b))}{[c_{ee}(\theta, e(\theta, b))]^3} \geq 0 \text{ by } (1);
\]
\[
e_{bb}(\theta, b) = -q_1 e(\theta, b) \frac{c_{\theta e}(\theta, e(\theta, b))}{[c_{ee}(\theta, e(\theta, b))]^2} \leq 0 \text{ by } (1);
\]
\[
S_\theta(\theta, b) = (q_1 - c_e(\theta, e(\theta, b))) e_\theta(\theta, b) - c_\theta(\theta, e(\theta, b));
\]
\[
S_b(\theta, b) = q_1 (1 - b) e_b(\theta, b) \geq 0;
\]
\[
S_{\theta b}(\theta, b) = q_1 (1 - b) e_{\theta b}(\theta, b) \geq 0;
\]
\[
S_{bb}(\theta, b) = -q_1 (e(\theta, b)) e_b(\theta, b) + q_1 e(\theta, b) (1 - b) e_{bb}(\theta, b) \leq 0;
\]
\[
V_\theta(\theta, b) = -c_\theta(\theta, b) > 0;
\]
\[
V_b(\theta, b) = q_0 + q_1 e(\theta, b);
\]
\[
V_{\theta b}(\theta, b) = q_1 e_\theta(\theta, b) = -q_1 \frac{c_{\theta e}(\theta, e(\theta, b))}{c_{ee}(\theta, e(\theta, b))} \geq 0;
\]
\[
V_{bb}(\theta, b) = q_1 e_b(\theta, b) \geq 0;
\]
\[
\pi_{bb}(\theta, b) = S_{bb}(\theta, b) - V_{bb}(\theta, b) \leq 0. \tag{13}
\]

B Proofs

B.1 Proof of Lemma 1

Proof that $r_H = 0$

Assume $r_H > 0$. Note that $(IC_H)$ is binding since otherwise the firm would reduce $r_H$:

\[
p_H V(H, b_H) + r_H = p_L V(H, b_L) + r_L. \tag{14}
\]

From (6), the firm’s payoff from the $H$-type is

\[
p_H (S(H, b_H) - V(H, b_H)) + (1 - p_H) W - r_H
\]
\[
= p_H (S(H, b_H) - W) + W - p_L V(H, b_L) - r_L.
\]

Take a pair $(\Delta_r, \Delta_b) \in \mathbb{R}^2_+$ such that offering $r_H - \Delta_r$ and $b_H + \Delta_b$ keeps the equality (14):

\[
p_H V(H, b_H + \Delta_b) + r_H - \Delta_r = p_L V(H, b_L) + r_L. \tag{15}
\]
Offering \( r_H - \Delta_r \) and \( b_H + \Delta_b \) improves the firm’s payoff since the social welfare is improved by (13) and the rent paid for the \( H \)-type is fixed by (15). Moreover, \((IC_L)\) is satisfied:

\[
p_H V (L, b_H + \Delta_b) + r_H - \Delta_r = p_H V (L, b_H) + p_H V (L, b_H + \Delta_b) - p_H V (L, b_H) - \Delta_r \\
\leq p_H V (L, b_L) + r_L + p_H V (L, b_H + \Delta_b) - p_H V (L, b_H) - \Delta_r
\]

by \((IC_L)\) for the original contract

\[
\leq p_H V (L, b_L) + r_L + p_H V (H, b_H + \Delta_b) - p_H V (H, b_H) - \Delta_r.
\]

The last line follows since \( V_{\theta b} (\theta, b) \geq 0 \) by (13):

\[
p_H V (H, b_H + \Delta_b) - p_H V (H, b_H) = p_H V (L, b_H + \Delta_b) - p_H V (L, b_H) + p_H \int_{\theta = L}^{H} (V_{\theta} (\theta, b_H + \Delta_b) - V_{\theta} (\theta, b_H)) d\theta
\]

\[
= p_H V (L, b_H + \Delta_b) - p_H V (L, b_H) + p_H \int_{\theta = L}^{H} \int_{b = b_H}^{b_H + \Delta_b} V_{\theta b} (\theta, b) d\Delta_b d\theta
\]

\[
\geq p_H V (L, b_H + \Delta_b) - p_H V (L, b_H).
\]

By (15), this inequality implies

\[
p_H V (L, b_H + \Delta_b) + r_H - \Delta_r \leq p_L V (L, b_L) + r_L.
\]

Therefore, we have \( r_H = 0 \).

**Proof that either \( p_L = 1 \) or \( p_H = 1 \)**

Assume next \( p_H < 1 \) and \( p_L < 1 \). From (6), the firm’s objective is

\[
\mu_H [p_H \pi (H, b_H) + (1 - p_H) W - r_H] + \mu_L [p_L \pi (L, b_L) + (1 - p_L) W - r_L]
\]

\[
= \mu_H [\pi (H, b_H) - W - r_H] + \mu_L [\pi (L, b_L) - W - r_L] + W.
\]

We have

\[
\mu_H [\pi (H, b_H) - W - r_H] + \mu_L [\pi (L, b_L) - W - r_L] > 0
\]

since otherwise \( p_H = p_L = r_H = r_L = 0 \) would be optimal (recall that we consider the case when \( \max \{p_H, p_L\} > 0 \)).

Offering \((kp_H, b_H, kr_H)\) and \((kp_L, b_L, kr_L)\) with \( k > 1 \) increases the firm’s profit:

\[
\mu_H [kp_H \pi (H, b_H) + (1 - kp_H) W - kr_H] + \mu_L [kp_L \pi (L, b_L) + (1 - kp_L) W - r_L]
\]

\[
= k \{\mu_H [p_H \pi (H, b_H) - W - r_H] + \mu_L [p_L \pi (L, b_L) - W - r_L]\} + W.
\]

Since \((IC_H)\) and \((IC_L)\) are satisfied, this is a profitable deviation, which is a contradiction.
B.2 Proof of Lemma 2

If \( r_L > 0 \) and \((IC_L)\) is not binding, then the firm would simply reduce \( r_L \) (regardless of Assumption 1). Suppose \( r_L > 0 \) and \((IC_H)\) is not binding. Then, reducing \( r_L > 0 \) and increasing \( b_L \) to keep \( p_L V(L, b_L) + r_L \) fixed improves the firm’s payoff and satisfies the incentive compatibility constraint since the social welfare is improved and the rent paid to the \( L \)-type is fixed, as in the proof of Lemma 1.

B.3 Proof of Lemma 3

If \( r_L = 0 \), then \((IC_H)\) implies \( p_H \geq p_L \) if \( b_H \leq b_L \), and \((IC_L)\) implies \( p_L \geq p_H \) if \( b_H \geq b_L \). If \( r_L > 0 \), then both \((IC_H)\) and \((IC_L)\) bind from Lemma 2. Hence
\[
\begin{align*}
  p_H V(H, b_H) &= p_L V(H, b_L) + r_L; \\
  p_H V(L, b_H) &= p_L V(L, b_L) + r_L,
\end{align*}
\]
and so
\[
\frac{p_H}{p_L} = \frac{V(H, b_L) - V(L, b_L)}{V(H, b_H) - V(L, b_H)}.
\]
Hence, \( p_H \geq p_L \) if and only if
\[
(V(H, b_L) - V(H, b_H)) - (V(L, b_L) - V(L, b_H)) \geq 0.
\]
Since
\[
(V(H, b_L) - V(H, b_H)) - (V(L, b_L) - V(L, b_H)) = \int_{\theta=L}^{H} \int_{b=b_H}^{b_L} V_{\theta b}(\theta, b) \, db \, d\theta
\]
and \( V_{\theta b}(\theta, b) = q_1 e_{\theta}(\theta, b) > 0 \) by (13), we have \( p_H \geq p_L \) if \( b_H \leq b_L \), and \( p_L \geq p_H \) if \( b_H \geq b_L \).

B.4 Proof of Proposition 1

\( p_H = 1 \) and \( p_L = 0 \) follows immediately from Assumption 1. Replacing \( p_H \) and \( p_L \) in (6) and solving for the firm’s problem without the (IC) constraints immediately yields \( b_H > 0 \) and \( b_L = r_L = r_H = 0 \).

B.5 Proof of Proposition 2

The firm’s problem is to choose \( \{p_{\theta}, b_{\theta}\}_{\theta \in \{L,H\}} \) to solve the following program:
\[
\max \mu_H [p_H \cdot (q_H \cdot (1 - b_H) - W) - r_H] + \mu_L [p_L \cdot (q_L \cdot (1 - b_L) - W) - r_L]
\]
subject to

\[ p_H \cdot q_H \cdot b_H + r_H \geq p_H \cdot q_H \cdot b_L + r_L; \]
\[ p_H \cdot q_L \cdot b_H + r_H \leq p_L \cdot q_L \cdot b_L + r_L. \]

The constraints represent the incentive compatibility constraints for types \( H \) and \( L \), respectively. In this problem, it follows immediately that \( p_H = 1 \) and \( p_L = b_H = r_H = r_L = 0 \) yields the highest payoff to the firm. The optimal contract for the firm is to assign only the \( H \)-type employee to the high-impact project and to make no payments to the employee.

### B.6 Proof of Proposition 3

The firm’s problem with \( W = 0 \) is

\[
\max_{p_H, p_L, b_H \geq 0, b_L \geq 0, r_L \geq 0} \mu_H p_H \pi(H, b_H) + \mu_L [p_L \pi(L, b_L) - r_L]
\]

subject to

\[
IC_H : p_H V(H, b_H) \geq p_L V(H, b_L) + r_L,
\]
\[
IC_L : p_L V(L, b_L) + r_L \geq p_H V(L, b_H),
\]

where the constraints incorporate the result of Lemma 1 that \( r_H = 0 \) for any value of \( W \).

To prove the result, by Lemma 3, it suffices to show that \( p_H = p_L = 1 \).

Case 1. Suppose both \( IC_H \) and \( IC_L \) are not binding. Then, the optimal contract would be \( \theta = \arg \max_\theta q(e(\theta, b))(1 - b) \) and \( p_H = p_L = 1 \).

Case 2. If \( IC_H \) is not binding but \( IC_L \) is binding, then \( p_L = 1 \) and \( r_L = 0 \) since Lemma 2 holds for any value of \( W \). Hence, \( IC_L \) implies

\[
V(L, b_L) = p_H V(L, b_H)
\]

\[
= p_H V(L, b_H) + (1 - p_H) V(L, 0) \geq V(L, p_H b_H)
\]

since \( V \) is convex in \( b \) by (13). Suppose instead the firm offers \( C_H = (1, p_H b_H, 0) \) to the \( H \)-type. \( IC_L \) holds by \( (IC_L) \). At the same time, since \( \pi \) is concave in \( b \) by (13), we have

\[
\pi(H, p_H b_H) \geq p_H \pi(H, b_H) + (1 - p_H) \pi(H, 0) \geq p_H \pi(H, b_H).
\]

Hence, this contract is profit improving. Hence, \( p_H = p_L = 1 \).

Note that the last inequality in (16) uses the fact that \( \pi(H, 0) \geq 0 \). With \( W > 0 \), we would have \( \pi(H, 0) - W \) instead, which could be negative, and the conclusion might not hold.

Case 3. If \( IC_L \) is not binding but \( IC_H \) is binding, then \( p_H = 1 \) and \( r_L = 0 \). Hence, \( IC_H \) is written as \( p_L V(H, b_L) = V(H, b_H) \). Suppose instead the firm offers \( C_L = (1, p_L b_L, 0) \) to the \( L \)-type. The same proof as above implies that this contract is incentive compatible and
profit improving. Hence, \( p_H = p_L = 1 \).

Case 4. Suppose both \( IC_H \) and \( IC_L \) are binding. Since Lemma 3 holds for any value of \( W \), we have \([1 = p_H \geq p_L \text{ and } b_H \leq b_L] \) or \([p_H \leq p_L = 1 \text{ and } b_H \geq b_L] \). Suppose \( 1 = p_H \geq p_L \) and \( b_H \leq b_L \). Since \( V(L,b_H) = p_L V(L,b_L) + r_L \geq V(L,p_L b_L) + r_L \) implies \( b_H \geq p_L b_L \), we have

\[
\pi(L,b_H) = S(L,b_H) - V(L,b_H) \\
\geq S(L,b_H) - p_L V(L,b_L) - r_L \text{ by } IC_L \\
\geq S(L,p_L b_L) - p_L V(L,b_L) - r_L \text{ by } b_H \geq p_L b_L \\
\geq p_L S(L,b_L) + S(L,0) - p_L V(L,b_L) - r_L \text{ by concavity of } S \text{ in } b \\
\geq p_L \pi(L,b_L) - r_L.
\]

Hence, it is profit-improving to offer \((p,b,r) = (p_H,b_H,0)\) to both players.

Suppose next \( p_H \leq p_L = 1 \) and \( b_H \geq b_L \). By the symmetric proof, if \( r_L = 0 \), then we have \( b_L \geq p_H b_H \);\(^{22}\) and if \( b_L \geq p_H b_H \), then it is profitable to offer \((p,b,r) = (p_L, b_L, 0)\) to both players. Hence, it remains to consider the case in which \( r_L > 0 \) and \( b_L \leq p_H b_H \). Suppose the firm offers \((p,b,r) = (1, p_H b_H, 0)\) to both players. Since \( S \) is concave in \( b \) and \( V \) is convex in \( b \), the profit from the \( H \)-type increases:

\[
S(H,p_H b_H) - V(H,p_H b_H) \\
\geq p_H S(H,b_H) + (1 - p_H) \underbrace{S(H,0) - p_H V(H,b_H)}_{\geq 0} - (1 - p_H) \underbrace{V(H,0)}_{=0} \\
\geq p_H S(H,b_H) - p_H V(H,b_H).
\]

The profit from the \( L \)-type also increases since

\[
\pi(L,p_H b_H) = S(L,p_H b_H) - V(L,p_H b_H) \\
\geq S(L,b_L) - V(L,p_H b_H) \text{ since } b_L \leq p_H b_H \\
\geq S(L,b_L) - p_H V(L,b_H) - (1-p_H) \underbrace{V(L,0)}_{=0} \text{ since } V \text{ is convex in } b \\
= S(L,b_L) - V(L,b_L) - r_L \text{ by } IC_L \\
= \pi(L,b_L) - r_L.
\]

Hence, it is profit-improving to offer \((p,b,r) = (1, p_H b_H, 0)\) to both players.

### B.7 Proof of Proposition 4

Given Lemma 1, it suffices to prove that \( p_L = 0 \) is optimal if and only if \( S(L,1) - W \leq 0 \).

\(^{22}\)The difference from the previous case is that, since \( v_L > 0 \), \( V(H,w_L) + v_L = p_H V(H,w_H) \geq V(L,p_H w_H) \) does not imply \( w_L \geq p_H w_H \).
The proof of $p_L = 0$ implying $S(L, 1) - W \leq 0$: Suppose $p_L = 0$ in the optimal contract. Then, we have

$$r_L = p_H V(L, b_H) \text{ since } IC_L \text{ binds}$$

$$< p_H V(H, b_H).$$

(17)

Hence, $IC_H$ is not binding.

Since $(IC_L)$ binds, it follows that $r_L = p_H V(L, b_H) - p_L V(L, b_L)$. Substituting $r_L$ into the firm’s problem,

$$\max_{p_H, p_L, b_H, b_L} \mu_H p_H [q(e(H, b_H))(1 - b_H) - W]$$

$$+ \mu_L [p_L q(e(L, b_L))(1 - b_L) - p_L W - r_L]$$

$$\Leftrightarrow$$

$$\max_{p_H, b_H} \mu_H p_H [q(e(H, b_H))(1 - b_H) - W] - p_H \mu_L V(L, b_H)$$

$$+ \mu_L \max_{p_L} \max_{b_L} [q(e(L, b_L)) - c(L, e(L, b_L)) - W].$$

(18)

Only if $\max_{b_L} [q(e(L, b_L)) - c(L, e(L, b_L)) - W] \leq 0$, the optimal $p_L$ is equal to 0.

The proof of $S(L, 1) - W \leq 0$ implying $p_L = 0$: Guess that $IC_H$ is slack. As above, the firm’s problem is equivalent to (18). If $S(L, 1) - W \leq 0$, then the optimal $p_L$ is equal to 0. Again, with $p_L = 0$, $IC_L$ implies $IC_H$ as (17). Hence, the guess of $IC_H$ being slack is verified.

### B.8 Complementary Lemmas

In the following lemma, we show that the condition for MRS stability is the property of log-submodularity / log-supermodularity of the relevant payoff function.

**Lemma 5** The employee’s MRS is decreasing (increasing) if and only if $V(\theta, b)$ is log-submodular (log-supermodular) in $(\theta, b)$. The firm’s MRS is decreasing (increasing) if and only if $S(\theta, b) - W$ is log-submodular (log-supermodular) in $(\theta, b)$.

The following lemma pins down the shape of the optimal contract given log-submodularity (or log-supermodularity) of $V$ and a binding constraint $IC_L$:

**Lemma 6** If $(IC_L)$ holds with equality, then,

1. If $V$ is log-submodular, then $b_H > b_L$ and $p_H < p_L$ if $r_L > 0$; and $b_H \leq b_L$ and $p_H \geq p_L$ if $r_L = 0$.

2. If $V$ is log-supermodular, then $b_H < b_L$ and $p_H > p_L$ if $r_L > 0$; and $b_H \geq b_L$ and $p_H \leq p_L$ if $r_L = 0.$
B.8.1 Proofs of Complementary Lemmas

Proof of Lemma 5  By definition, a function $F(\theta, b)$ is log-supermodular (log-submodular) if the following property holds:

$$F(\theta, b) F(\theta', b') - F(\theta', b) F(\theta, b') \geq (\leq) 0$$

if and only if $(\theta - \theta')(b - b') \geq (\leq) 0$. (19)

If log $F(\theta, b)$ is twice-differentiable, then it is log-supermodular (log-submodular) whenever $\frac{d^2}{dbdb} \log F(\theta, b) \geq (\leq) 0$.

For any arbitrary contract $C_H$, consider the set of contracts $C$ with $r = 0$ that would make type $\theta$ indifferent between $C_H$ and $C$. On the indifference curve obtained in this way, the marginal rate of substitution of $b$ for $p$ satisfies

$$V(\theta, b) dp + pV_b(\theta, b) db = 0 \iff -\frac{dp}{db} = \frac{V_b(\theta, b)}{V(\theta, b)}.$$

Then, $(-\frac{dp}{db})|_{\theta=H} \leq (\geq) (-\frac{dp}{db})|_{\theta=L}$ is equivalent to $p\frac{V_b(\theta, b)}{V(\theta, b)}|_{\theta=H} \leq (\geq) p\frac{V_b(\theta, b)}{V(\theta, b)}|_{\theta=L}$. With $V(\theta, b)$ differentiable with respect to $\theta$ and $b$, this implies $\frac{\partial V(\theta, b)}{\partial \theta} \leq (\geq) 0$. Thus we have

$$\frac{\partial V_b(\theta, b)}{\partial \theta} = \frac{V_{bb}(\theta, b)V(\theta, b) - V_b(\theta, b)V_b(\theta, b)}{(V(\theta, b))^2} = \frac{d^2}{dbdb} \log V(\theta, b) \leq (\geq) 0.$$

The analysis for $S(\theta, b) - W$ is analogous.

Proof of Lemma 6  We present the proof focusing on log-submodular $V$, since the argument for log-supermodular $V$ is symmetric.

Proof. Part 1: If $r_L > 0$, then $b_H > b_L$ and $p_H < p_L$. Since Lemma 2 implies both $(IC_L)$ and $(IC_H)$ are binding, rearranging them yields

$$\begin{cases} p_H = p_L \frac{V(H, b_L) - V(L, b_L)}{V(H, b_H) - V(L, b_H)}, \\ p_H V(L, b_H) = p_L V(L, b_L) + r_L. \end{cases}$$

(20)

Since $r_L > 0$, we have $p_H V(L, b_H) > p_L V(L, b_L)$, and so

$$\frac{V(H, b_L) - V(L, b_L)}{V(H, b_H) - V(L, b_H)} V(L, b_H) > V(L, b_L) \iff V(H, b_L) V(L, b_H) > V(H, b_H) V(L, b_L).$$

Hence we have $b_H > b_L$ if $V$ is log-submodular. (20) implies $p_H < p_L$.

Part 2: If $r_L = 0$, then $b_H \leq b_L$ and $p_L \geq p_H$. Since $(IC_L)$ is satisfied with equality, it follows that $p_H V(L, b_H) = p_L V(L, b_L)$. If $V$ is log-submodular and $b_H > b_L$, then $V(H, b_H) / V(L, b_H) < V(H, b_L) / V(L, b_L)$, and so $p_H V(H, b_H) < p_L V(H, b_L)$. This con-
trades to \((IC_H)\). Hence \(b_H \leq b_L\), and so \(p_H/p_L = V(L,b_L)/V(L,b_H) \geq 1\). ■

B.9 Proof of Proposition 5

If it offers \(C_H\) to the \(L\)-type, the firm’s profit changes by

\[
0 \geq [p_H \pi(L,b_H) + (1-p_H)W] - [p_L \pi(L,b_L) - r_L + (1-p_L)W]
\]
\[
\geq p_H [S(L,b_H) - W] - p_L [S(L,b_L) - W] \text{ by } (IC_L).
\] (21)

On the other hand, if it offers \(C_L\) to the \(H\)-type, its profit changes by

\[
0 \geq [p_L \pi(H,b_L) - r_L + (1-p_L)W] - [p_H \pi(H,b_H) + (1-p_H)W]
\]
\[
\geq p_L [S(H,b_L) - W] - p_H [S(H,b_H) - W] \text{ by } (IC_H).
\]

Rearranging, we obtain

\[
p_H [S(H,b_H) - S(L,b_H)] \geq p_L [S(H,b_L) - S(L,b_L)].
\]

Since (21) implies \(p_H \leq p_L [S(L,b_L) - W]/[S(L,b_H) - W]\), we obtain

\[
\frac{S(L,b_L) - W}{S(L,b_H) - W} \left[ (S(H,b_H) - W) - (S(L,b_H) - W) \right]
\]
\[
\geq \left[ (S(H,b_L) - W) - (S(L,b_L) - W) \right]
\]
\[
\Leftrightarrow (S(L,b_L) - W)(S(H,b_H) - W) \geq (S(L,b_H) - W)(S(H,b_L) - W).
\]

Hence if the net social welfare is log-supermodular (or log-submodular), then \(b_H \geq b_L\) (or \(b_H \leq b_L\)). Given Lemma 6 and \(V(\theta,b)\) log-submodular, it follows that \(r_L > 0\) if and only if \(b_H > b_L\).

B.10 Proof of Corollary 1

Log-submodularity (or log-supermodularity) of \(F\) is equivalent to

\[
\frac{d^2 \log F(\theta,b)}{dbd\theta} \leq (\geq) 0.
\]

By the envelope theorem, the sign of \(\frac{d^2 \log V(\theta,b)}{dbd\theta}\) is determined by

\[
q_1 \epsilon_\theta(\theta,b)V(\theta,b) - [q_0 + q_1 \epsilon(\theta,b)] V_\theta(\theta,b)
\] (22)
and that of \( \frac{d^2 \log(S(\theta, b) - W)}{db^2} \) is determined by

\[
q_1 (1 - b) e_\theta (\theta, b) [S(\theta, b) - W] - q_1 (1 - b) e_\theta (\theta, b) S_\theta (\theta, b). \tag{23}
\]

Notice that \( V_\theta (\theta, b) = -c_\theta (\theta, b) \) and \( e_\theta (\theta, b) = -c_\theta e (\theta, e(b)) e_{ee}(\theta, e(b)) \) by (13). Thus, \( V_\theta (\theta, b) \) is not directly affected by \( e_\theta (\theta, b), \) and so (22) increases in \( e_\theta (\theta, b). \)

The function \( S_\theta (\theta, b) \) is an increasing function of \( e_\theta (\theta, b) \) by (13), so (23) decreases in \( e_\theta (\theta, b). \) Moreover, (23) increases in \( e_{\theta \theta} (\theta, b) \). The conclusion about the sufficient magnitudes of \( e_\theta (\theta, b) \) and \( e_{\theta \theta} (\theta, b) \) follows.

### B.11 Proof of Proposition 6

By Proposition 5, we have \( r_L = 0 \) and \( b_H \leq b_L. \) Lemmas 1 and 6 imply that \( p_H = 1. \) Since \((IC_L)\) is satisfied with equality by Assumption 1, we have \( V(L, b_H) = p_L V(L, b_L). \) Since \( V \) is log-submodular and \( b_H \leq b_L, \) we have \( V(H, b_H)/V(L, b_H) \geq V(H, b_L)/V(L, b_L), \) and so \( V(H, b_H) \geq p_L V(H, b_L). \) Hence \((IC_H)\) is satisfied whenever \( b_H \leq b_L \) and \((IC_L)\) holds with equality. Thus, the firm’s problem becomes

\[
\max_{b_H, b_L, b_H \geq b_H \geq 0} \mu_H (\pi (H, b_H) - W) + \mu_L p_L (\pi (L, b_L) - W)
\]

subject to \( V(L, b_H) = p_L V(L, b_L). \) Substituting the constraint yields (11).

### B.12 Proof of Proposition 7

By Proposition 5, we have \( b_H \geq b_L. \) Lemmas 1 and 6 imply that \( p_L = 1. \)

Suppose \( b_H \neq b_L, \) but \((IC_H)\) holds with strict inequality (and so \( r_L = 0). \) Since \((IC_L)\) is satisfied with equality, we have \( p_H V(L, b_H) = V(L, b_L). \) Since \( V \) is log-submodular and \( b_H > b_L, \) we have \( V(H, b_H)/V(L, b_H) < V(H, b_L)/V(L, b_L), \) and so \( p_H V(H, b_H) < p_L V(H, b_L). \) Hence \((IC_H)\) is not satisfied. Thus, \( b_H \neq b_L \) implies \((IC_H)\) holds with equality. In addition, \( b_H = b_L \) implies \( p_H = p_L \) and \( r_L = 0 \) from Lemma 6. Hence, regardless of \( (b_H, b_L), (IC_H) \) holds with equality. Substituting \((IC_H)\) and \((IC_L)\) into (6) yields (12). Moreover, rearranging \((IC_H)\) and \((IC_L)\), yields

\[
r_L = \frac{V(H, b_L) V(L, b_H) - V(L, b_L) V(H, b_H)}{V(H, b_H) - V(L, b_H)};
\]

which is positive if and only if \( b_H > b_L \) given \( V \) log-submodular.