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B Three-Period Model for Comparative Statics

In this appendix, we introduce the three-period model to theoretically derive comparative statics. As mentioned in the main text, effort choice is binary, \( e \in \{0, 1\} \) with \( c(0) = 0 \) and \( c(1) = \gamma \). We write \( \Pr(\omega|e) = \alpha_\omega + \beta_\omega e \) for each \( \omega \). We also write \( \beta_G = \beta_G + \beta_{bG} \), \( \beta_b = \beta_{bG} + \beta_{bB} \), and so on. We keep Assumption 2: \( \beta_{gG}, \beta_{bG} > 0 \), \( \beta_{bB}, \beta_{bB} < 0 \), and \( \alpha_{bG} = 0 \). We also assume that \( \beta_{bB}/\alpha_{bB} < \beta_{gB}/\alpha_{gB} \) (hazard rate is lower for \( bB \)). Given this assumption, starting from a fixed initial belief, the belief update satisfies

\[
\mu_{bB} < \mu_{gB} < \mu < \mu_{gB} < \mu_{bG} = 1.
\]

(86)

We assume that keeping the agent for another term after \( \omega \in \{gG, bG\} \) or keeping him for another two terms after \( \omega \in \{gG\} \) is sufficient to incentivize the effort: \( \gamma < \min \left\{ \delta \beta_{gG}, \delta \beta_{gG} \left( 1 + \delta - \frac{1-\alpha_{gG}}{\beta_{gG}} \gamma \right) \right\} \), where the term \( \frac{1-\alpha_{gG}}{\beta_{gG}} \gamma \) represents the cost of effort that the agent pays to exert effort in the second period.

We assume that there are only three periods. In period 1, the principal has initial belief \( \mu_1 \in [0, 1] \) and starts with the initial promised value \( V_1 \in [1, 1 + \delta + \delta^2] \), and the principal cannot replace the agent in period 1 (that is, \( V_1 \) is the promised value conditional on the realization of the public randomization in the language of the infinitely repeated game). Although the game formally starts from period 1, by endowing the principal with \( (\mu_1, V_1) \), we can measure the effect of the state variable \( (\mu_1, V_1) \).

In periods 2 and 3, for simplicity, we assume that the principal cannot observe a signal \( s \). Hence, the principal’s payoff is \(-C\) if \( y = B \) and 0 otherwise.\(^{20}\) As in the main text, in periods 2 and 3, the newly arriving agent is an \( H \)-type with probability \( \mu_H \).

B.1 Backward Induction

We now specify the principal’s optimal strategy by backward induction.

Period 3 In the third period, since there is no future period, no agent exerts effort. Hence, regardless of \( (\mu, V) \), the principal’s payoff is \( J_3(\mu, V) = \alpha_B (-C) \).

\(^{20}\) This assumption keeps the derivation of the continuation payoff simple and allows us to talk about the main trade-off faced by the principal in a clear way. If we allowed her to observe a warning, then the principal could tailor the intervention decision based on her belief and promised value in period 2. This additional effect would increase the principal’s incentive to learn the agent’s type, so not to intervene in period 1.
Period 2  If the public randomization tells her to keep the agent and provide him with value \( V \), the principal solves (ignoring the continuation payoff since it is constant)

\[
J^*_2(\mu, V) = \max_{e \in \{0,1\}, V_e \in [0,\delta]} \mu_H u^P(e) + (1 - \mu_H) u^P(0)
\]

subject to

\[
\gamma \leq \delta \sum_{\omega} \beta_\omega V_\omega \text{ if } e = 1, \quad (87)
\]

\[
V = 1 - \gamma \cdot 1_{\{e=1\}} + \sum_{\omega} (\alpha_\omega + \beta_\omega e) V_\omega.
\]

(PK)

Suppose the promise keeping (PK) is not binding. Since \( u^P(e) \) is increasing in \( e \) and \( e = 1 \) is implementable, we have

\[
J^*_2(\mu) = - (\alpha_B + \mu \beta_B) C.
\]

(89)

We now derive the range of \( V \) in which PK is not binding. Effort \( e = 1 \) is implementable if and only if \( \beta_G \delta V_G + \beta_B \delta V_B \geq \gamma \), and given \( e = 1 \), the agent obtains \( 1 - \gamma + (\alpha_G + \beta_G) \delta V_G + (\alpha_B + \beta_B) \delta V_B \). Given \( \gamma < \delta \beta_G \), the lowest payoff that the agent obtains given \( e = 1 \) is

\[
\bar{V}_2 := 1 + \gamma \alpha_G / \beta_G \text{ and the highest payoff is}
\]

\[
\bar{V}_2 := 1 - \gamma + (\alpha_G + \beta_G) \delta + (\alpha_B + \beta_B) \frac{\gamma - \delta \beta_G}{\beta_B} = 1 + \delta - \frac{1 - \alpha_G}{\beta_G} \gamma,
\]

(90)

where the second equality follows from \( \alpha_G + \alpha_B = 1 \) and \( \beta_G + \beta_B = 0 \).

Given this observation, we solve for \( J^*_2(\mu, V) \). If

\[
\frac{J^*_2(\mu_H) + \alpha_B C}{1 + \delta} \geq \frac{J^*_2(\mu) + \alpha_B C}{1 + \delta - \bar{V}_2} \implies \mu \leq \mu := \mu_H \frac{1}{1 + \delta} - \frac{1 - \alpha_G}{\beta_G} \gamma,
\]

(91)

then the principal maximizes the probability of replacing the current agent by mixing \( V = 0 \) and \( V = 1 + \delta \). Hence,

\[
J^*_2(\mu, V) = \frac{1 + \delta - V}{1 + \delta} J^*(\mu_H) - \frac{V}{1 + \delta} \alpha_B C \text{ for each } V \in [0,1 + \delta].
\]

(92)

If \( \mu \in [\mu_H, \mu] \), then since the problem is linear, it is optimal to mix 0 and \( \bar{V}_2 \) for \( V \leq \bar{V}_2 \) and to mix \( \bar{V}_2 \) and \( 1 + \delta \) otherwise. Hence, we have

\[
J^*_2(\mu, V) = \begin{cases} 
\frac{\bar{V}_2 - V}{\bar{V}_2} J^*_2(\mu_H) + \frac{V}{\bar{V}_2} J^*_2(\mu) & \text{for } V \leq \bar{V}_2, \\
\frac{1 + \delta - V}{1 + \delta - \bar{V}_2} J^*(\mu) - \frac{V - \bar{V}_2}{1 + \delta - \bar{V}_2} \alpha_B C & \text{for } V > \bar{V}_2.
\end{cases}
\]

(93)

Finally, for \( \mu \geq \mu_H \), for \( V < \bar{V}_2 \), it is optimal to mix 0 and \( \bar{V}_2 \); and for \( V > \bar{V}_2 \), it is
optimal to mix $\bar{V}_2$ and $1 + \delta$. Hence,

$$J_2^* (\mu, V) = \begin{cases} \frac{V - V_2}{\bar{V}_2} J_2^* (\mu_H) + \frac{V}{\bar{V}_2} J_2^* (\mu) & \text{for } V < V_2, \\ \frac{1 + \delta - V}{1 + \delta - \bar{V}_2} J_2^* (\mu) - \frac{V - V_2}{1 + \delta - \bar{V}_2} \alpha BC & \text{for } V > \bar{V}_2. \end{cases}$$

(94)

**Period 1** In the first period, the principal given $(\mu_1, V_1)$ maximizes

$$J_1 (\mu_1, V_1) = \max_{\iota \in \{0, 1\}, e \in \{0, 1\}, V_\omega \in [0, 1 + \delta]} \mu_1 u^P(\iota, e) + (1 - \mu_1) u^P(\iota, 0)$$

$$+ \delta \sum_\omega (\alpha_\omega + \mu_1 \beta_\omega e) J_2 (\mu_\omega, V_\omega)$$

(95)

subject to

$$\mu_\omega = \frac{\mu_1 (\alpha_\omega + \beta_\omega)}{\alpha_\omega + \mu_1 \beta_\omega};$$

$$\gamma \leq \delta \sum_\omega \beta_\omega V_\omega \text{ if } e = 1,$$

$$V_1 = 1 - \gamma \cdot 1_{\{e = 1\}} + \delta \sum_\omega (\alpha_\omega + \beta_\omega e) V_\omega.$$  

(96)

Except for the claim about $V_1$ in Proposition 6, we assume that PK is not binding. Hence, we omit $V_1$ until we prove Proposition 6. If $e = 0$, then $\iota = 1$ is optimal since there is no learning or incentive reason not to intervene. Since the continuation payoff is concave in $V_\omega$, it is optimal to have $V_\omega = \frac{1}{\delta} (V_1 - 1)$ and obtain

$$-\alpha_b \iota - \alpha gBC + \delta J_2^* \left( \mu_1, \frac{1}{\delta} (V_1 - 1) \right).$$

(97)

Especially, if PK is not binding, then the principal’s value is

$$-\alpha_b \iota - \alpha gBC + \delta J_2^* \left( \max \{\mu_1, \mu_H\} \right).$$

(98)

If $e = 1$ is implemented, then we can write the problem as

$$J_1 (\mu_1) = \max_{\iota, V_\omega} \mu u^P(\iota, 1) + (1 - \mu) u^P(\iota, 0) + \delta \sum_\omega (\alpha_\omega + \mu \beta_\omega) J_2 (\mu_\omega, V_\omega)$$

(99)

subject to

$$\mu_\omega = \frac{\alpha_\omega + \beta_\omega}{\alpha_\omega + \mu_1 \beta_\omega} \text{ and } \gamma \leq \delta \sum_\omega \beta_\omega V_\omega.$$  

(100)

Since the derivation of the optimal continuation payoffs requires tedious algebra, we first summarize the result: For $\iota = 0$, there are following four cases, depending on the value of $\mu_1$ and $\mu_H$:
1. $\mu_H < \mu_{B} < \mu_{G} < \mu_{BG}$: $V_{BG} = V_{G} = \bar{V}$, and find the maximum $V_{B} \in [0, \bar{V}]$ and $V_{bB} \in [0, \bar{V}]$ to satisfy the incentive compatibility constraint (IC), $\gamma \leq \delta \sum_{\omega} \beta_{\omega} V_{\omega}$. We first decrease $V_{bB}$ before we decrease $V_{B}$: $V_{bB} < \bar{V}$ only if $V_{bB} = 0$. The principal’s payoff is

$$J_1(\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C) + \delta \alpha_B (-C) + \delta \mu_1 \beta_B (-C)$$

$$- \frac{\delta \beta_\omega (V_2 - V_\omega)}{V_2} \left( \frac{V_\omega}{\beta_\omega} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C. \quad (101)$$

2. $\mu_{B} < \mu_H < \mu_G < \mu_{BG}$: $V_{BG} = V_{G} = \bar{V}$, $V_{bB} = 0$, and find the maximum $V_{B} \in [0, \bar{V}]$ to satisfy the incentive compatibility constraint (IC), $\gamma \leq \delta \sum_{\omega} \beta_{\omega} V_{\omega}$. The principal’s payoff is

$$J_1(\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C)$$

$$+ \delta \alpha_B (\alpha_G + \mu_1 \beta_G) (-C) + \delta \beta_B \mu_1 (\alpha_G + \beta_G) (-C)$$

$$+ \delta \alpha_B (\alpha_B + \mu_1 \beta_B) (-C) + \delta \beta_B \mu_1 (\alpha_B + \beta_B) (-C) \quad (102)$$

$$+ \delta (\alpha_{BG} + \mu_1 \beta_{BG}) (\alpha_B + \mu_H \beta_B) (-C)$$

$$- \frac{\delta \beta_{BG} (V_2 - V_\omega)}{V_2} \left( \frac{V_\omega}{\beta_{BG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C.$$

3. $\mu_{BG} < \mu_{B} < \mu_H < \mu_{BG}$: $V_{BG} = V_{G} = \bar{V}$, and $V_{bB} = V_{BG} = 0$. The principal’s payoff is

$$J_1(\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C)$$

$$+ \delta \alpha_B (\alpha_G + \mu_1 \beta_G) (-C) + \delta \beta_B \mu_1 (\alpha_G + \beta_G) (-C)$$

$$+ \delta (\alpha_B + \mu_1 \beta_B) (\alpha_B + \mu_H \beta_B) (-C). \quad (103)$$

4. $\mu_{BG} < \mu_{B} < \mu_G < \mu_H < \mu_{BG}$: $V_{BG} = \bar{V}$ and $V_{bB} = V_{BG} = 0$, and find the minimum $V_{G} \in [0, \bar{V}]$ to satisfy IC. The principal’s payoff is

$$J_1(\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C)$$

$$+ \delta \alpha_B (\alpha_{BG} + \mu_1 \beta_{BG}) (-C) + \delta \beta_B \mu_1 (\alpha_{BG} + \beta_{BG}) (-C)$$

$$+ \delta (\alpha_B + \mu_1 \beta_B) (\alpha_B + \mu_H \beta_B) (-C) \quad (104)$$

$$+ \frac{\delta \beta_{BG} V_U}{V_2} \left( \frac{V_U}{\beta_{BG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C.$$

For $\ell = 1$, there are following four cases, depending on the value of $\mu_1$ and $\mu_H$:

1. $\mu_H < \mu_B, \mu_{BG} < \mu_G$: $V_{G} = \bar{V}$, and find the maximum $V_{bB} \in [0, \bar{V}]$ and $V_B \in [0, \bar{V}]$ to
satisfy IC. We first decrease $V_\omega$ with lower $\frac{\beta_\omega}{\omega}$: $V_{\omega^*} < V_2$ for $\omega^* = \arg \max_{\omega \in \{b, gB\}} \frac{\beta_\omega}{\omega}$ only if $V_{\omega^*} = 0$ for $\omega^* = \arg \min_{\omega \in \{b, gB\}} \frac{\beta_\omega}{\omega}$.

The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_b + \mu_1 \beta_b) (-l) + (\alpha_{gB} + \mu_1 \beta_{gB}) (-C) + \delta \alpha_B (-C) + \delta \mu_1 \beta_B (-C)$$

$$- \sum_{\omega \in \{b, gB\}} \frac{\delta \beta_\omega (V_2 - V_\omega)}{V_2} \left( \frac{\alpha_\omega}{\beta_\omega} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C. \quad (105)$$

2. $\min \{\mu_b, \mu_{gB}\} < \mu_H < \max \{\mu_b, \mu_{gB}\} < \mu_{gG}$: $V_{bG} = V_{gG} = \bar{V}_2$, $V_{\omega^*} = 0$ for $\omega^* = \arg \min_{\omega \in \{b, gB\}} \frac{\beta_\omega}{\omega}$, and find the maximum $V_{\omega^*} \in [0, V_2]$ to satisfy IC for $\omega^{**} = \arg \max_{\omega \in \{b, gB\}} \frac{\beta_\omega}{\omega}$.

The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_b + \mu_1 \beta_b) (-l) + (\alpha_{gB} + \mu_1 \beta_{gB}) (-C) + \delta \alpha_B (\alpha_{gG} + \alpha_{\omega^*} + \mu_1 (\beta_{gG} + \beta_{\omega^*})) (-C) + \delta \beta_B \mu_1 (\alpha_{gG} + \beta_{gG}) (-C) + \delta (\alpha_{gG} + \alpha_b + \mu_1 (\beta_{gB} + \beta_b)) (\alpha_B + \mu_H \beta_B) (-C)$$

$$- \frac{\delta \beta_{\omega^*} (V_2 - V_{\omega^*})}{V_2} \left( \frac{\alpha_{\omega^*}}{\beta_{\omega^*}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C. \quad (106)$$

3. $\max \{\mu_b, \mu_{gB}\} < \mu_H < \mu_{gG}$: $V_{bG} = \bar{V}_2$ and $V_{gB} = V_b = 0$. The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_b + \mu_1 \beta_b) (-l) + (\alpha_{gB} + \mu_1 \beta_{gB}) (-C) + \delta \alpha_B (\alpha_{gG} + \mu_1 \beta_{gG}) (-C) + \delta \beta_B \mu_1 (\alpha_{gG} + \beta_{gG}) (-C) + \delta (\alpha_{gB} + \alpha_b + \mu_1 (\beta_{gB} + \beta_b)) (\alpha_B + \mu_H \beta_B) (-C). \quad (107)$$

4. $\mu_{gG} < \mu_H$: $V_{gB} = V_b = 0$, and find the minimum $V_{gG} \in [0, V_2]$ to satisfy IC. The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_b + \mu_1 \beta_b) (-l) + (\alpha_{gB} + \mu_1 \beta_{gB}) (-C) + \delta (\alpha_{gG} + \mu_1 \beta_{gG}) (\alpha_B + \mu_H \beta_B) (-C) + \delta (\alpha_{gB} + \alpha_b + \mu_1 (\beta_{gB} + \beta_b)) (\alpha_B + \mu_H \beta_B) (-C)$$

$$+ \frac{\delta \beta_{gG} V_{gG}}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C. \quad (108)$$

### B.2 Period-1 Problem without Promise Keeping

Suppose $\iota = 0$ is optimal. Given (86), there are following four cases:
1. \( \mu_H < \mu_{bb} < \mu_{gB} < \mu_{gg} < \mu_{bb} \): In this case, the principal would like to set \( V_{gg} = V_{bb} = V_2 \). This satisfies IC if and only if \( \delta \beta_{gg}V_2 + \delta \beta_{bb}V_2 \geq \gamma \).

If IC is not satisfied, then the principal either increases \( V_{gg} \) or \( V_{bb} \), or she decreases \( V_{gB} \) or \( V_{bB} \). To relax the IC constraint by one unit by changing \( V! \), we must change \( V! \) by \( \frac{1}{\delta \beta_!} \). The marginal effect of increasing \( V_{gg} \) by \( \frac{1}{\delta \beta_{gg}} \) units on \( \alpha \sum_\omega (\alpha_\omega + \mu_1 \beta_\omega)J_2(\mu_\omega, V_\omega) \) is

\[
\frac{1}{\beta_{gg}}(\alpha_{gg} + \mu_1 \beta_{gg}) \frac{\mu_{gg}\beta_{bb}C}{1 + \delta - V_2} = \mu_1 \left( \frac{\alpha_{gg}}{\beta_{gg}} + 1 \right) \frac{\beta_{bb}C}{1 + \delta - V_2} < 0. \tag{109}
\]

Similarly, the marginal payoff of increasing \( V_{bb} \) by \( \frac{1}{\delta \beta_{bb}} \) is \( \mu_1 \frac{\beta_{bb}C}{1+\delta-V_2} < 0 \).

The cost to change \( V_{bb} \) by \( \frac{1}{\delta \beta_{bb}} \) is

\[
\frac{1}{\beta_{bb}}(\alpha_{bb} + \mu_1 \beta_{bb}) \frac{J^*_2(\mu_{bb}) - J^*_2(\mu_H)}{V_2} = \frac{1}{V_2} \left( \frac{\alpha_{bb}}{\beta_{bb}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_{bb}C. \tag{110}
\]

Similarly, the cost to change \( V_{bb} \) by \( \frac{1}{\delta \beta_{bb}} \) is \( \frac{1}{V_2} \left( \frac{\alpha_{bb}}{\beta_{bb}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_{bb}C \).

Since \( 1 + \delta - V_2 < \delta < 1 < V_2 \), and \( \frac{\alpha_{gg}}{\beta_{gg}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \in [0, \mu_1] \) given \( \mu_\omega \geq \mu_H \), it is optimal to decrease \( V_{bb} \) or \( V_{bb} \) instead of increasing \( V_{gg} \) or \( V_{bb} \).

Moreover, given that \( \frac{\beta_{bb}}{\alpha_{bb}} < \frac{\beta_{bb}}{\alpha_{gg}} \), we have \( 0 < \frac{\alpha_{bb}}{\beta_{bb}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) < \frac{\alpha_{gg}}{\beta_{gg}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \), so it is most efficient to decrease \( V_{bb} \) then decrease \( V_{gB} \).

If decreasing \( V_{bb} \) is enough, that is, if \( \delta \beta_{gg}V_2 + \delta \beta_{bb}V_2 \geq \gamma \), then the principal decreases \( V_{bb} \) such that IC holds with equality: \( \delta \beta_{gg}V_2 + \delta \beta_{bb}V_2 + \delta \beta_{bb}V_{bb} = \gamma \). Otherwise, she sets \( V_{bb} = 0 \) and decreases \( V_{gB} \) such that IC is satisfied with equality: \( \delta \beta_{gg}V_2 + \delta \beta_{bb}V_{bb} = \gamma \). Since \( V_{bb} = 0 \) satisfies IC by assumption, there exists \( V_{gB} \in [0, V_2] \) to satisfy IC for sure.

In total, we have \( V_{bb} = V_{gg} = V_2 \), and largest \( V_{gB} \in [0, V_2] \) and \( V_{bb} \in [0, V_2] \) to satisfy
IC $\gamma \leq \delta \sum_\omega \beta_\omega V_\omega$. Moreover, $V_gB < V_2$ only if $V_{bB} = 0$. The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C) + \delta \sum_{\omega \in \{bG, gG\}} (\alpha_\omega + \mu_1 \beta_\omega) J_2^* (\mu_\omega) + \delta \sum_{\omega \in \{bB, gB\}} (\alpha_\omega + \mu_1 \beta_\omega) \left( \frac{V_2 - V_\omega}{V_2} J_2^* (\mu_H) + \frac{V_\omega}{V_2} J_2^* (\mu_\omega) \right) \quad (111)$$

$$= (\alpha_B + \mu_1 \beta_B) (-C) + \delta \alpha_B (-C) + \delta \mu_1 \beta_B (-C) - \sum_{\omega \in \{bB, gB\}} \frac{\delta \beta_\omega (V_2 - V_\omega)}{V_2} \left( \frac{\alpha_\omega}{\beta_\omega} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C.$$

2. $\mu_{bB} < \mu_H < \mu_{gB} < \mu_{gG} < \mu_{bG}$: In this case, the principal would like to set $V_gG = V_{bG} = \bar{V}_2$, $V_gB = \bar{V}_2$, and $V_{bB} = 0$. This satisfies IC if and only if $\delta \beta_{gG} \bar{V}_2 + \delta \beta_{gB} \bar{V}_2 \geq \gamma$.

If this condition is not satisfied, then the principal either increases $V_{gG}$ or $V_{bG}$, or decreases $V_{gB}$. As in Case 1, it is optimal to decrease $V_{gB}$ instead of increasing $V_{gG}$ or $V_{bG}$. Hence, the optimal continuation payoff will be: $V_{gG} = V_{bG} = \bar{V}_2$, $V_{bB} = 0$, and $V_{gB} \geq 0$ solves IC with equality: $\delta \beta_{gG} \bar{V}_2 + \delta \beta_{gB} V_{gB} = \gamma$.

Hence, we have $V_{bG} = V_{gG} = \bar{V}_2$, $V_{bB} = 0$, and largest $V_{gB} \in [0, \bar{V}_2]$ to satisfy IC $\gamma \leq \delta \sum_\omega \beta_\omega V_\omega$. The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C) + \delta \alpha_B (\alpha_G + \mu_1 \beta_G) (-C) + \delta \beta_{bB} \mu_1 (\alpha_G + \beta_G) (-C) + \delta \alpha_B (\alpha_{gB} + \mu_1 \beta_{gB}) (-C) + \delta \beta_{bB} \mu_1 (\alpha_{gB} + \beta_{gB}) (-C) + \delta (\alpha_{bB} + \mu_1 \beta_{bB}) (\alpha_B + \mu_H \beta_B) (-C) - \frac{\delta \beta_{gB} (V_2 - V_\omega)}{V_2} \left( \frac{\alpha_{gB}}{\beta_{gB}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C. \quad (112)$$

3. $\mu_{bB} < \mu_{gB} < \mu_H < \mu_{gG} < \mu_{bG}$: In this case, the principal would like to set $V_{gG} = V_{bG} = \bar{V}_2$, $V_gB = V_{bB} = 0$. This satisfies IC by assumption. The principal’s payoff is

$$J_1 (\mu_1) = (\alpha_B + \mu_1 \beta_B) (-C) + \delta \alpha_B (\alpha_G + \mu_1 \beta_G) (-C) + \delta \beta_{bB} \mu_1 (\alpha_G + \beta_G) (-C) + \delta (\alpha_{bB} + \mu_1 \beta_{bB}) (\alpha_B + \mu_H \beta_B) (-C). \quad (113)$$

4. $\mu_{bB} < \mu_{gB} < \mu_H < \mu_{bG}$: In this case, the principal would like to set $V_{bG} = \bar{V}_2$, $V_gG = V_gB = V_{bB} = 0$. This satisfies IC if and only if $\delta \beta_{bG} \bar{V}_2 \geq \gamma$.

Otherwise, the principal has to either increase $V_{bG}$ or increase $V_{gG}$. On the one hand,
the marginal payoff of increasing \( V_{gG} \) by \( \frac{1}{\beta_{gG}} \) units on \( \delta \sum_{\omega}(\alpha_{\omega} + \mu_1 \beta_{\omega})J_2(\mu_{\omega}, V_{\omega}) \) is, if \( \mu_{gG} > \underline{\mu} \), then

\[
\frac{1}{\beta_{gG}}(\alpha_{gG} + \mu_1 \beta_{gG}) \frac{J^*_g(\mu_{gG}) - J^*_h(\mu_H)}{V_2} = \frac{1}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_{B}C, \tag{114}
\]

and if \( \mu_{gG} < \underline{\mu} \), then

\[
\mu_1 \left( \frac{\alpha_{gG}}{\beta_{gG}} + 1 \right) \frac{\beta_{B}C}{1 + \delta}. \tag{115}
\]

By definition of \( \underline{\mu} \), the cost of increasing \( V_{gG} \) is no more than

\[
\frac{1}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_{B}C. \tag{116}
\]

On the other hand, the marginal payoff to increase \( V_{gG} \) by \( \frac{1}{\delta \beta_{gG}} \) is

\[
\mu_1 \frac{\beta_{B}C}{1 + \delta - V_2}. \tag{117}
\]

Since \( 1 + \delta - V_2 \leq \delta, V_2 \geq 1, \) and \( \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \in [0, \mu_1] \) given \( \mu_{gG} \leq \mu_H \), it is optimal to increase \( V_{gG} \) such that IC holds with equality. Hence, the optimal continuation payoff will be: \( V_{gG} = V_2, V_{gB} = V_b = 0, \) and smallest \( V_{gG} \geq 0 \) to satisfy \( \delta \beta_{gG}V_2 + \delta \beta_{gG}V_{gG} \geq \gamma \). The principal’s payoff is

\[
J_1(\mu_1) = (\alpha_B + \mu_1 \beta_B)(-C) + \delta \alpha_B(\alpha_{gG} + \mu_1 \beta_{gG})(-C) + \delta \beta_B \mu_1 (\alpha_{gG} + \beta_{gG})(-C) + \delta (\alpha_{gG} + \mu_1 \beta_{gG})(\alpha_B + \mu_H \beta_B)(-C) + \delta (\alpha_B + \mu_1 \beta_B)(\alpha_B + \mu_H \beta_B)(-C) + \frac{\delta \beta_{gG}V_{gG}}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_{B}C. \tag{118}
\]

Suppose next that \( \iota = 1 \) is optimal. Let \( \omega^* \in \{b, gB\} \) and \( \omega^{**} \in \{b, gB\} \) such that \( \mu_{\omega^*} < \mu_{\omega^{**}} \). There are following four possible cases:

1. \( \mu_H < \mu_{\omega^*} < \mu_{\omega^{**}} < \mu_{gG} \): In this case, the principal would like to set \( V_{gG} = V_2, V_{gB} = V_b = V_2 \). This satisfies IC if and only if

\[
\delta \beta_{gG} V_2 + \delta (\beta_{gB} + \beta_B) V_2 \geq \gamma. \tag{119}
\]

If this condition is not satisfied, then the principal either increases \( V_{gG} \), or decreases
V_b or V_{gB}. On the one hand, the marginal payoff of increasing V_{gG} by \( \frac{1}{\delta_{gG}} \) is

\[
\mu_1 \left( \frac{\alpha_{gG}}{\beta_{gG}} + 1 \right) \frac{\beta_{BC}}{1 + \delta - V_2}.
\]

On the other hand, for \( \omega \in \{b, gB\} \), the cost to change \( V_\omega \) by \( \frac{1}{\delta_{\omega}} \) is

\[
\frac{1}{\beta_\omega} \left( \alpha_\omega + \mu_1 \beta_\omega \right) \frac{J^*_2(\mu_\omega) - J^*_2(\mu_H)}{V_2} = \frac{1}{\beta_\omega} \left( \frac{\alpha_\omega}{\beta_\omega} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \frac{\beta_{BC}}{1 + \delta - V_2}.
\]

Since \( 1 + \delta - V_2 < \delta < V_2 \) and \( \frac{\alpha_\omega}{\beta_\omega} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \in [0, \mu_1] \), it is optimal to decrease \( V_{gB} \) or \( V_b \) instead of increasing \( V_{gG} \). Moreover, the marginal payoff of changing \( V_\omega^* \) by \( \frac{1}{\delta_{\omega}} \) is higher. Hence, the optimal continuation payoff will be: \( V_{gG} = \bar{V}_2, \) \( V_{\omega^*} = [0, \bar{V}_2] \), and \( V_{\omega**} = V_2 \) if there exists \( V_{\omega^*} \in [0, \bar{V}_2] \) to solve IC with equality: \( \delta_{gG} \bar{V}_2 + \delta_{\omega} \bar{V}_2 = \gamma \). Otherwise, \( V_{gG} = \bar{V}_2, \) \( V_{\omega^*} = 0, \) and \( V_{\omega**} \in [0, \bar{V}_2] \), where \( V_{\omega**} \in [0, \bar{V}_2] \) solves IC with equality: \( \delta_{gG} \bar{V}_2 + \delta_{\omega} \bar{V}_2 = \gamma \).

The calculation of the principal’s payoff is the same as in the case with \( \iota = 0 \), so it is omitted.

2. \( \mu_{\omega^*} < \mu_H < \mu_{\omega**, < \mu_{gG}} \): In this case, the principal would like to set \( V_{gG} = \bar{V}_2, \) \( V_{\omega**} = \bar{V}_2, \) and \( V_{\omega^*} = 0 \). If this is not enough, then by the same calculation as Case 1, it is more efficient to decrease \( V_{\omega**} \). Hence we take \( V_{\omega**} \in [0, \bar{V}_2] \) to solve IC when it holds with equality:

\[
\delta_{gG} \bar{V}_2 + \delta_{\omega^*} \bar{V}_2 = \gamma.
\]

3. \( \mu_{\omega^*} < \mu_{\omega**, < \mu_H} < \mu_{gG} \): In this case, the principal would like to set \( V_{gG} = \bar{V}_2 \) and \( V_{\omega**} = \bar{V}_2, \) and \( V_{\omega^*} = 0 \). By definition, this continuation payoff satisfies IC.

4. \( \mu_{\omega^*} < \mu_{\omega**, < \mu_{gG}} < \mu_H \): As in Case 4 for \( \iota = 0 \), the principal sets \( V_{gG} \in [0, 1 + \delta] \) and \( V_{\omega**} = \bar{V}_2, \) and \( V_{\omega^*} = 0 \), where \( V_{gG} \) solves IC with equality: \( \delta_{gG} \bar{V}_2 = \gamma \).

**B.3 Proof of Proposition 4**

**Discount Factor \( \delta \):** Let \( J_1(\iota, \delta) \) be the principal’s payoff given \( e = 1, \iota \in \{0, 1\} \), and \( \delta \in [0, 1] \), keeping all the other parameters fixed. We show that the marginal benefit of no intervention is increasing in \( \delta \in [0, 1] \):

\[
\frac{d}{d\delta} (J_1(0, \delta) - J_1(1, \delta)) \geq 0.
\]

Note that the following variables are independent of \( \delta \): \( J^*_2(\cdot), \bar{V}_2, \) and \( 1 + \delta - \bar{V}_2 \). Given \( e = 1, \) let \( \omega(\iota) \) be the outcome \( \omega \) such that \( V_\omega \) changes as we change \( \delta \), and let \( w(\iota, \delta) := \)
\[ \delta \sum_{\omega \neq \omega(t)} \beta_{\omega} V_\omega(t) \] be the expected sum of the other continuation payoffs. Note that \( \omega(t) \) is non-empty if and only if IC is binding.

Suppose that it is not the case that \( \beta_{\omega(t)} = g_G \) and \( g_G < \mu \). Note that \( \delta \beta_{\omega(t)} V_{\omega(t)}(t) + w(t, \delta) = \gamma \) if IC is binding, and so \( \frac{d}{d\delta} \left( \delta \beta_{\omega(t)} V_{\omega(t)}(t) \right) = -\sum_{\omega \neq \omega(t)} \beta_{\omega} V_\omega(t) \). As seen in Appendix B.2, the marginal effect of changing \( V_{\omega(t)}(t) \) by \( 1/(\delta \beta_{\omega(t)}) \) unit (or changing \( \delta \beta_{\omega(t)} V_{\omega(t)} \) by one unit) is independent of \( \delta \). Hence, there exists \( mc_{\omega(t)}(\delta) \geq 0 \) such that the marginal effect of increasing \( \delta \) is given by

\[
\frac{d}{d\delta} J_1(t, \delta) = mc_{\omega(t)}(\delta) + J_2(t, \delta),
\]

where \( J_2(t, \delta) \) is the expected continuation payoff from period 2. In particular, \( mc_{\omega(t)}(\delta) \) corresponds to the absolute value of the marginal payoff of increasing \( \delta \beta_{\omega(t)} V_{\omega(t)} \) by one unit, multiplied by \( \sum_{\omega \neq \omega(t)} \beta_{\omega} V_\omega(t) \) (the change in \( \delta \beta_{\omega(t)} V_{\omega(t)}(t) \) when we change \( \delta \)).

Since \( J_2 \) is piecewise linear and no intervention guarantees the higher continuation payoff for the principal, the benefit in terms of continuation payoff from no intervention is larger than the cost that the principal has to pay in state \( \omega(t) \):

\[
mc_{\omega(1)}(\delta) - mc_{\omega(0)}(\delta) \leq J_2(0, \delta) - J_2(1, \delta).
\]

Hence, we have the desired inequality (123).

Next, suppose \( \mu_G < \mu < \mu_H \). If \( \omega(1) = \emptyset \) (IC is not binding for intervention), then we have \( \omega(0) = \emptyset \) since no intervention allows the principal to monitor the effort more precisely. Hence, (123) holds.

If \( \omega(0) = \omega(1) = g_G \), then for \( t = 0 \), we have \( w(t, \delta) = 0 \) and \( \delta \beta_{gG} V_{gG} \) is fixed at \( \gamma - \delta \beta_{bG} V_2 \). The expected continuation payoff from \( V_{gG} \), \( \delta (\alpha_{gG} + \mu_1 \beta_{gG}) J_2(\mu_G, V_{gG}) \), is

\[
\delta (\alpha_{gG} + \mu_1 \beta_{gG}) \left( \frac{1 + \delta - V_{gG}}{1 + \delta} J^*(\mu_H) - \frac{V_{gG}}{1 + \delta} \alpha_B C \right) = \delta (\alpha_{gG} + \mu_1 \beta_{gG}) (\alpha_B + \mu_H \beta B) (-C) \]

\[- \frac{1}{1 + \delta} \frac{\gamma}{\beta_{gG}} (\alpha_{gG} + \mu_1 \beta_{gG}) \mu_H \beta B (-C) + \delta \frac{\beta_{gG} V_2}{1 + \delta} (\alpha_{gG} + \mu_1 \beta_{gG}) \mu_H \beta B (-C).
\]

For \( t = 1 \), in contrast, \( \delta \beta_{gG} V_{gG} \) is fixed at \( \gamma \). The expected continuation payoff from \( V_{gG}, \)
\[ \delta(\alpha_G + \mu_1 \beta_G)J_2(\mu_G, V_G), \]
is
\[ \delta \left( \alpha_G + \mu_1 \beta_G \right) \left( \frac{1 + \delta - V_G J^*(\mu_H) - V_{\beta G}}{1 + \delta} \alpha_B C \right) \]
\[ = \delta \left( \alpha_G + \mu_1 \beta_G \right) (\alpha_B + \mu_H \beta_B) (-C) \]
\[ - \frac{1}{1 + \Delta \beta_G} \left( \alpha_G + \mu_1 \beta_G \right) \mu_H \beta_B (-C) \tag{127} \]

since \( \delta (\alpha_\omega + \mu_1 \beta_\omega) J_2 (\mu_\omega, V_\omega) = \delta J_2 (\mu_H) \) for each \( \omega \neq gG \). Direct calculation implies (123).

The proof for \( \omega(0) = \emptyset \) and \( \omega(1) = gG \) is analogous.

**Cost of Effort** \( \gamma \): Let \( J_1 (\iota, \gamma) \) be the principal’s payoff given \( \iota = 1, \ \iota \in \{0, 1\} \), and \( \gamma \), keeping all the other parameters fixed. There exists a set of parameters such that

\[ d \frac{d}{d\gamma} (J_1(0, \gamma) - J_1(1, \gamma)) \tag{128} \]
is negative for some \( \gamma \) while it is positive for others.

For example, suppose that, with \( \iota = 0, \mu_H < \mu_B < \mu_{gB} < \mu_{bG} \), and \( V_{gB} = V_2 \) and \( V_{bB} \in (0, V_2) \) to satisfy IC:

\[ \delta (\beta_{BG} + \beta_{GB}) V_2 + \delta \beta_{BB} V_2 + \delta \beta_{BB} V_B = \gamma. \tag{129} \]

At the same time, assume that, with \( \iota = 1, \mu_H < \mu_B < \mu_{gB} < \mu_{gG} \), and \( V_{gB} = V_2 \) and \( V_B \in (0, V_2) \) to satisfy IC:

\[ \delta \beta_{BG} V_2 + \delta \beta_{BB} V_2 + \delta \beta_{BB} V_B = \gamma. \tag{130} \]

Since \( \frac{d}{d\gamma} V_2 = -\frac{1-\alpha_{GB}}{\beta_G}, \beta_{BG} > 0, \beta_B < 0, \) and \( \beta_{BB} < 0 \), given (129) and (130), we have \( \frac{d}{d\gamma} \beta_{BB} V_B > \frac{d}{d\gamma} \beta_B V_B \). Since \( \mu_H < \mu_{BB}, \mu_B, \) increasing \( \beta_{BB} V_B \) and \( \beta_B V_B \) (decreasing \( V_B \) and \( V_B \)) is costly. In particular, depending on the relative values of \( \frac{d}{d\gamma} \beta_{BB} V_B, \frac{d}{d\gamma} \beta_B V_B, \frac{d}{d\gamma} (\alpha_{BB} + \beta_{BB} \delta J_2 (\mu_{BB}, V_{BB}), ) \) and \( \frac{d}{d\gamma} (\alpha_B + \beta_{B} \delta J_2 (\mu_B, V_B)), \frac{d}{d\gamma} (J_1(0, \gamma) - J_1(1, \gamma)) \) can be negative or positive. A specific numerical example is available upon request.

**B.4 Proof of Proposition 5**

For \( \iota = 0, \) since the instantaneous payoff \( -(\alpha_B + \beta_B) C \) stays the same, we focus on the continuation payoff. Except for Case 4 of Appendix B.2, the continuation payoff is constant given \( \beta_B \) and \( \beta_G \) being fixed.

In Case 4, \( V_{bG} = V_2, V_{gB} = V_{bB} = 0, \) and we set \( V_{gG} \in [0, \bar{V}_2] \) as the smallest value to satisfy IC: \( \delta \beta_{BG} V_2 + \delta \beta_{BB} V_G = \gamma. \) Hence, the marginal effect of increasing \( \beta_{BG} \) (and decreasing \( \beta_{BB} \)) on \( \delta \beta_{BG} V_G \) is \( -\delta V_2. \) The marginal effect of increasing \( \beta_{BG} \) (and decreasing
\( \beta_{gG} \) on the principal’s payoff is, given the result of Appendix B.2,

\[
\begin{align*}
&\delta \alpha_B \mu_1 (-C) + \delta \beta_B \mu_1 (-C) - \delta \mu_1 (\alpha_B + \mu_H \beta_B) (-C) \\
+ &\frac{-\delta V_2}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C \\
+ &\frac{\delta \beta_{gG} V_{gG}}{V_2^2} \left( \frac{\alpha_{gG}}{\beta_{gG}} \right)^2 (\mu_H - \mu_1) \beta_B C \\
= &\delta \frac{\alpha_{gG}}{\beta_{gG}} (\mu_H - \mu_1) (-\beta_B) C \left( 1 - \frac{V_{gG}}{V_2} \right). 
\end{align*}
\]

Since \( \mu_{gG} < \mu_H \) implies \( \mu_H - \mu_1 > 0 \) and we have \( V_{gG} < V_2 \), the marginal payoff is positive.

For \( \iota = 1 \), the instantaneous payoff \(- (\alpha_b + \beta_b) t - (\alpha_{bB} + \beta_{bB}) C \) decreases since \( \beta_b \) increases while other variables stay the same. In addition, the continuation payoff decreases as well. Since the verification is analogous in all the four cases listed in Appendix B.2, here, we focus on Case 1: \( \mu_H < \mu_b, \mu_{gB} < \mu_{gG} \). Recall that \( V_{gG} = V_2 \), and we take the largest \( V_{gB} \in [0, V_2] \) and \( V_b \in [0, V_2] \) to satisfy IC: \( \delta \beta_{gG} V_2 + \delta \beta_{gB} V_{gB} + \delta \beta_b V_b \geq \gamma \). Hence, \( \delta \beta_{gB} V_{gB} \) and \( \delta \beta_b V_b \) increase as \( \beta_{gG} \) decreases: \( \frac{\delta \beta_{gB} V_{gB}}{\delta \beta_{gG}} - \frac{\delta \beta_b V_b}{\delta \beta_{gG}} \geq 0 \). Given the result of Appendix B.2, the total effect on the continuation payoff is

\[
\begin{align*}
&\sum_{\omega \in \left\{ (b, bB) \right\}} \left( \frac{d \delta \beta_{\omega} V_{\omega}}{d \beta_{gG}} - \frac{d \delta \beta_{\omega} V_{\omega}}{d \beta_{gG}} \right) \frac{1}{V_2} \left( \frac{\alpha_{\omega}}{\beta_{\omega}} (\mu_H - \mu_1) - \mu_1 (1 - \mu_H) \right) \beta_B C \\
+ &\frac{\delta (V_2 - V_b)}{V_2} \frac{\alpha_b}{\beta_b} (\mu_H - \mu_1) \beta_B C < 0. \quad \text{for one period is not sufficient to incentivize the effort. Given this assumption, since } \alpha_{gG} > 0, \text{ to implement } e = 1, \text{ the } H \text{-type agent obtains the payoff more than one (that is, the principal has to pay the rent to the agent). Hence, with } V_1 = 1, \text{ the principal has to implement } e = 0. \text{ So, optimal } \iota \text{ is } 1. \text{ Similarly, with } V_1 = 1 + \delta + \delta^2, \text{ the principal has to retire the agent, so } e = 0 \text{ and intervention is optimal.}

On the other hand, suppose \( V_1 \in [1, 1 + \delta + \delta^2] \) is such that PK is not binding. Then, the results in Appendix B.2 implies that \( e = 1 \) and \( \iota = 0 \) are optimal for sufficiently small \( \beta_{bB} \).

**Initial Belief** \( \mu_1 \): We consider non-binding PK, and let \( J_1(\mu_1, \iota) \) be the principal’s payoff given \( e = 1, \iota \in \{0, 1\} \), and \( \mu_1 \in [0, 1] \), keeping all the other parameters fixed. Suppose the \( \mu_1 \) is sufficiently small such that \( \mu_{gG} < \mu_H \). In addition, the reward \( V_2 \) after \( \omega = bG \) is
sufficient to satisfy IC with \( V_{gG} = V_{gB} = V_{bB} = 0 \). Then, \( \frac{d}{d\mu} J_1 (\mu, 0) \) is equal to

\[
\begin{align*}
\beta_B (-C) \\
+ \delta \alpha_B \beta_{bG} (-C) + \delta \beta_B (\alpha_{bG} + \beta_{bG}) (-C) \\
+ \delta \beta_{gG} (\alpha_B + \mu_H \beta_B) (-C) \\
+ \delta \beta_B (\alpha_B + \mu_H \beta_B) (-C),
\end{align*}
\]

while \( \frac{d}{d\mu} J_1 (\mu, 1) \) is equal to

\[
\begin{align*}
\beta_b (-l) + \beta_{gB} (-C) \\
+ \delta \beta_{gG} (\alpha_B + \mu_H \beta_B) (-C) \\
+ \delta (\beta_{gB} + \beta_b) (\alpha_B + \mu_H \beta_B) (-C) \\
+ \frac{\gamma}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} (-1) - (1 - \mu_H) \right) \beta_B C.
\end{align*}
\]

Hence,

\[
\frac{d}{d\mu} J_1 (\mu, 0) - \frac{d}{d\mu} J_1 (\mu, 1)
= \beta_{bG} l - \beta_{bB} (C - l) + \delta |\beta_B| \left( \beta_{bG} (1 - \mu_H) - \frac{\gamma}{V_2} \left( \frac{\alpha_{gG}}{\beta_{gG}} + (1 - \mu_H) \right) \right). \tag{135}
\]

At the limit where \( \delta \to 1 \) and \( |\delta \beta_{bG} V_2 - \gamma| \to 0 \), this value is

\[
\beta_{bG} \left( l - \frac{\alpha_{gG}}{\beta_{gG}} |\beta_B| C \right) - \beta_{bB} (C - l). \tag{136}
\]

This can be positive or negative, depending on the parameters. Hence, for sufficiently large \( \delta \) and small \( |\delta \beta_{bG} V_2 - \gamma| \), the sign of \( \frac{d}{d\mu} J_1 (\mu, 0) - \frac{d}{d\mu} J_1 (\mu, 1) \) is not determined.